Fuzzy & Annotated Semantic Web Languages

Umberto Straccia

ISTI - CNR, Pisa, Italy

umberto.straccia@isti.cnr.it
www.umbertostraccia.it

November 2018

About Vagueness

On the Existence of Vague Concepts

What are vague concepts and do they exists?

- Try to answer: What is this picture about?

(Registan Square, Samarkand, Uzbekistan)

- Vague concept: no unambiguous definition, e.g.
  - What is a picture or piece of text about ?
  - What is a tall person ?
  - What is a high temperature ?
  - What is nice weather ?
  - What is an adventurous trip ?
- Vague concepts:
  - Are abundant in everyday speech and almost inevitable
  - Their meaning is often subjective and context dependent

On the Existence of Vague Objects

What are vague objects and do they exists?

- Are there vague objects in the pictures?

(Erg Chebbi, pre-Sahara dunes, Merzouga, Morocco)

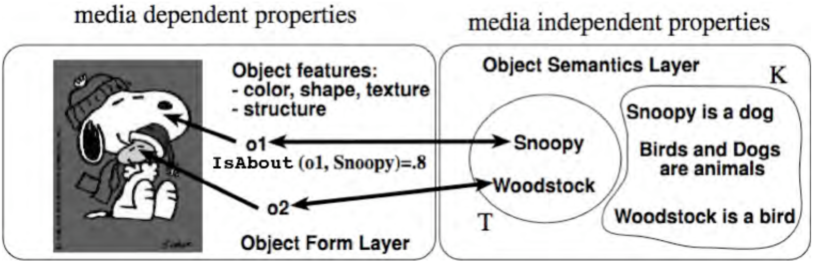
(The Sun)

- Vague object: its identity is lacking in clarity
  - Cloud
  - Dunes
  - Sun
- Vague objects:
  - Are not identical to anything, except to themselves (reflexivity)
  - Are characterised by a vague identity relation (e.g. a similarity relation)

Vague Statements

- A statement is vague whenever it involves vague concepts or vague objects
- The truth of a vague statement is a matter of degree,
  - it is intrinsically difficult to establish whether the statement is entirely true or false
  - The weather temperature is 33 °C. Is it hot?

Sources of Vagueness: Multimedia information retrieval



IsAbout		
ImageRegion	*Object ID*	*degree*
*o*1	*snoopy*	0.8
*o*2	*woodstock*	0.7
⋮	⋮	

"Find top-*k* image regions about animals"

$$Query(x) \leftarrow ImageRegion(x) \land isAbout(x, y) \land Animal(y)$$

Sources of Vagueness: Lifezone mapping

- To which degree do certain areas have a specific bioclima

Holdridge life zones of USA

Sources of Vagueness: ARPAT, Air quality in the province of Lucca

Sintesi dei dati rilevati dalle ore 0 alle ore 24 del giorno domenica 14/02/2010

Stazione		Tipo stazione	SO_2 µg/m³ (media su 24h)	NO_2 µg/m³ (max oraria)	CO mg/m³ (max oraria)	O_3 µg/m³ (max oraria)	PM_{10} µg/m³ (media su 24h)	Giudizio di qualità dell'aria
Lucca	P.za San Micheletto (RETE REGIONALE **)	urbana - traffico	1	75	---	---	56	Scadente
Lucca	V.le Carducci	urbana - traffico	2	---	2	---	75	Pessima
Lucca	Carignano (RETE REGIONALE **)	rurale - fondo	---	---	---	87 (h.18*)	---	Buona
Viareggio	Largo Risorgimento	urbana - traffico	---	---	1,7	---	n.d.	Buona
Viareggio	Via Maroncelli (RETE REGIONALE **)	urbana - fondo	1	121	---	60 (h.17*)	45	Accettabile
Capannori	V. di Piaggia (RETE REGIONALE **)	urbana - fondo	---	79	2	---	59	Scadente
Porcari	V. Carrara (RETE REGIONALE **)	periferica - fondo	2	72	---	82 (h.16*)	63	Scadente

Giudizio di qualità	SO_2 µg/m³ (media su 24h)	NO_2 µg/m³ (max oraria)	CO mg/m³ (max oraria)	O_3 µg/m³ (max oraria)	PM_{10} µg/m³ (media su 24h)
Buona	0-50	0-50	0-2,5	0-120	0-25
Accettabile	51-125	51-200	2,6-15	121-180	26-50
Scadente	126-250	201-400	15,1-30	181-240	51-74
Pessima	>250	>400	>30	>240	>74

http://www.arpat.toscana.it/

TripAdvisor: Hotel User Judgments

Uncertainty vs Vagueness: a clarification

- Initial difficulty:
  - Understand the conceptual differences between uncertainty and vagueness
- Main problem:
  - Interpreting a degree as a measure of uncertainty rather than as a measure of vagueness

Uncertain Statements

- A statement is true or false in any world/interpretation
  - We are "uncertain" about which world to consider as the actual one
  - We may have e.g. a probability/possibility distribution over possible worlds
- E.g., of uncertain statement: "it will rain tomorrow"
  - We cannot exactly establish whether it will rain tomorrow or not, due to our incomplete knowledge about our world
  - But, we may estimate to which degree this is e.g. probable/possible

Vague Statements

- A statement is vague if it involves vague concepts
- A statement is true to some degree, which is taken from a truth space (usually $[0, 1]$)
- E.g. of vague statement: “heavy rain”
  - is graded and the degree depends on the amount of rain is falling

In weather forecasts one may find:

Rain. Falling drops of water larger than 0.5 mm in diameter. “Rain” usually implies that the rain will fall steadily over a period of time;

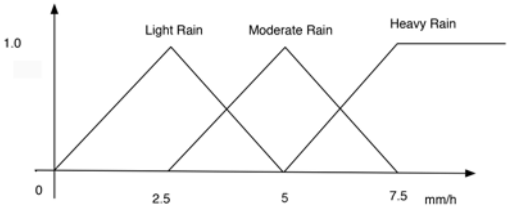
Light rain. Rain falls at the rate of 2.6 mm or less an hour;

Moderate rain. Rain falls at the rate of 2.7 mm to 7.6 mm an hour;

Heavy rain. Rain falls at the rate of 7.7 mm an hour or more.

- Quite harsh distinction:
$$\begin{array}{lcl} R = 7.7mm/h & \rightarrow & \text{heavy rain} \\ R = 7.6mm/h & \rightarrow & \text{moderate rain} \end{array}$$
- Unsatisfactory:
  - the more rain is falling, the more the sentence “heavy rain” is true
  - vice-versa, the less rain is falling the more the sentence “heavy rain” is false

- I.e., the sentence “heavy rain” is intrinsically graded
- More fine grained approach:
  - Define the various types of rains as



  - Light rain, moderate rain and heavy rain are vague concepts

- Are there sentences combining the two orthogonal concepts of uncertainty and vagueness?
- Yes, and we use them daily !
  - E.g. “*There will be heavy rain tomorrow*."
- This type of sentences are called uncertain vague sentences
- Essentially, there is
  - uncertainty about the world we will have tomorrow
  - vagueness about the various types of rain

From Fuzzy Sets to Mathematical Fuzzy Logic

Fuzzy Sets Basics

From Crisp Sets to Fuzzy Sets.

- Let X be a universal set of objects
- The crisp membership function of a set $A \subseteq X$:

  $$\chi_A \colon X \to \{0, 1\}$$

  where $\chi_A(x) = 1$ iff $x \in A$
- Fuzzy set A:

  $$\chi_A \colon X \to [0, 1]$$

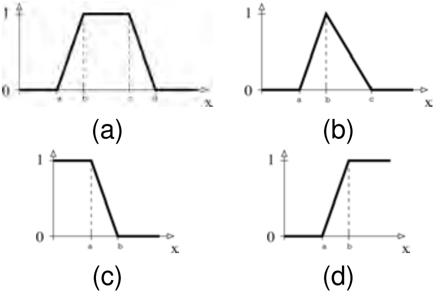
  or simply $A \colon X \to [0, 1]$
- Example: the fuzzy set

  $$C = \{x \mid x \text{ is a day with heavy precipitation rate } R\}$$

  is defined via the membership function

  $$\chi_C(x) = \begin{cases} 1 & \text{if } R \geq 7.5 \\ (x - 5)/2.5 & \text{if } R \in [5, 7.5) \\ 0 & \text{otherwise} \end{cases}$$

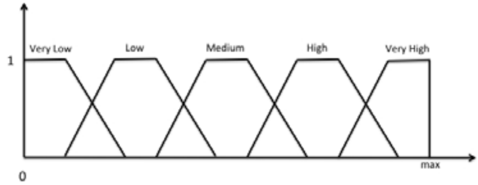
- Fuzzy membership functions may depend on the context and may be subjective
- Shape may be quite different
- Usually, it is sufficient to consider functions



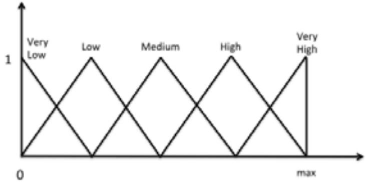
(a) Trapezoidal *trz*(*a*, *b*, *c*, *d*); (b) Triangular *tri*(*a*, *b*, *c*); (c) left-shoulder *ls*(*a*, *b*); (d) right-shoulder *rs*(*a*, *b*)

Fuzzy Sets Construction

- Simple and typically satisfactory method (numerical domain):
  - uniform partitioning into 5 fuzzy sets

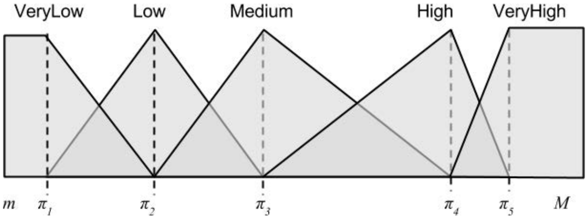


Fuzzy sets construction using trapezoidal functions



Fuzzy sets construction using triangular functions

- Another popular method is based on clustering
- Use Fuzzy C-Means to cluster data into 5 clusters
  - Fuzzy C-Means extends K-Means to accommodates graded membership
- From the clusters $c_1, \ldots, c_5$ take the centroids $\pi_1, \ldots, \pi_5$
- Build the fuzzy sets from the centroids



Fuzzy sets construction using clustering

Norm-Based Fuzzy Set Operations

- Standard fuzzy set operations are not the only ones
- Most notable ones are triangular norms
  - t-norm $\otimes$ for set intersection
  - t-conorm $\oplus$ (also called s-norm) for set union
  - negation $\ominus$ for set complementation
  - implication $\Rightarrow$ for set inclusion
- These functions satisfy some properties that one expects to hold

Łukasiewicz, Gödel, Product logic and Standard Fuzzy logic

- One distinguishes three different sets of fuzzy set operations (called fuzzy logics)
  - Łukasiewicz, Gödel, and Product logic
  - Standard Fuzzy Logic (SFL) is a sublogic of Łukasiewicz
    - $\min(a, b) = a \otimes_l (a \Rightarrow_l b)$, $\max(a, b) = 1 - \min(1 - a, 1 - b)$

	Łukasiewicz Logic	Gödel Logic	Product Logic	SFL
$a \otimes b$	$\max(a + b - 1, 0)$	$\min(a, b)$	$a \cdot b$	$\min(a, b)$
$a \oplus b$	$\min(a + b, 1)$	$\max(a, b)$	$a + b - a \cdot b$	$\max(a, b)$
$a \Rightarrow b$	$\min(1 - a + b, 1)$	$\begin{cases} 1 & \text{if } a \leq b \\ b & \text{otherwise} \end{cases}$	$\min(1, b/a)$	$\max(1 - a, b)$
$\ominus a$	$1 - a$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$\begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases}$	$1 - a$

- Mostert–Shields theorem: any continuous t-norm can be obtained as an ordinal sum of Ł, G and P.

Mathematical Fuzzy Logics Basics

- OWL 2 is grounded on Mathematical Logic
- Fuzzy OWL 2 is grounded on Mathematical Fuzzy Logic
- A statement is graded
- Truth space: set of truth values L
- Given a statement ϕ
  - Fuzzy Interpretation: a function $\mathcal{I}$ mapping ϕ into L, i.e.

$$\mathcal{I}(\varphi) \in L$$

  - Usually

$$\begin{array}{rcl} L & = & [0,1] \\ L_n & = & \{0, \frac{1}{n}, \ldots, \frac{n-2}{n-1}, \ldots, 1\} \quad (n \geq 1) \end{array}$$

- Fuzzy statement: for formula ϕ and $r \in [0, 1]$

$$\langle \phi, r \rangle$$

The degree of truth of ϕ is equal or greater than r

Fuzzy Semantic Web Languages and Beyond

The Semantic Web Family of Languages

- Wide variety of languages
  - RDFS: *Triple language*, *-Resource Description Framework*
    - The logical counterpart is ρdf
  - RIF: *Rule language*, *-Rule Interchange Format*,
    - Relate to the *Logic Programming* (LP) paradigm
  - OWL 2: *Conceptual language*, *-Ontology Web Language*
    - Relate to Description Logics (DLs)

RDFS

- RDFS: the triple language

$$\langle subject, predicate, object \rangle$$

e.g. $\langle umberto, born, zurich \rangle$

- OWL 2 family: an object oriented language

```
class      PERSON partial
           restriction (hasName someValuesFrom String)
           restriction (hasBirthPlace someValuesFrom GEOPLACE)
           ...
```

OWL 2 Profiles

OWL 2 EL

- Useful for large size of properties and/or classes
- The EL acronym refers to the $\mathcal{EL}$ family of DLs

OWL 2 QL

- Useful for very large volumes of instance data
- Conjunctive query answering via query rewriting and SQL
- OWL 2 QL relates to the DL family *DL-Lite*

OWL 2 RL

- Useful for scalable reasoning without sacrificing too much expressive power
- OWL 2 RL maps to Datalog

RIF/RuleML

- RIF/RuleML family: the rule language

  `Forall` ?Buyer ?Item ?Seller
  buy(?Buyer ?Item ?Seller) :- sell(?Seller ?Item ?Buyer)

Important point: RDFS, OWL 2 and RIF/RuleML are logical languages

- RDFS: logic with intensional semantics
- OWL 2: relates to the *Description Logics* family
- RIF/RuleML: relates to the *Logic Programming* paradigm (e.g., Datalog, Datalog$^{\pm}$)
- OWL 2 and RIF/RuleML have extensional semantics

The case of Fuzzy & Annotated RDFS

Fuzzy RDFS

- Triples may have attached a degree n in L or L_n

$$\langle(\mathit{subject}, \mathit{predicate}, \mathit{object}), n\rangle$$

- Meaning: the degree of truth of the statement is at least n
- Example:

$$\langle(o1, \mathit{IsAbout}, \mathit{snoopy}), 0.8\rangle$$

- How to represent fuzzy triples in RDFS?
  - Use reification method:

$$(s1, \mathit{hasObj}, o1), (s1, \mathit{hasRel}, \mathit{IsAbout}), (s1, \mathit{hasObj}, \mathit{snoopy}), (s1, \mathit{hasDeg} 0.8)$$

- Unfortunately, RDFS is lacking the "annotation property" of triples

Fuzzy RDFS Query Answering

- Conjunctive query: extends a crisp RDF query and is of the form

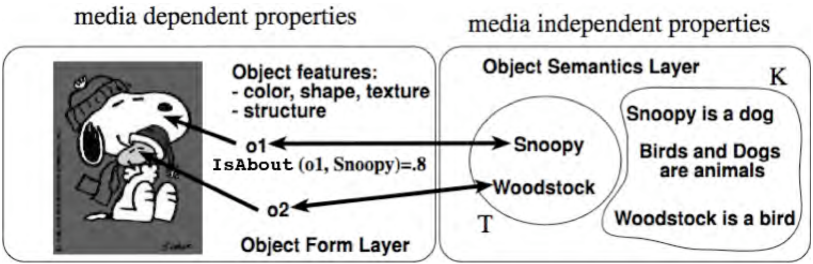
$$\begin{array}{rcl}\langle q(\mathbf{x}), s\rangle & \leftarrow & \exists \mathbf{y}.\langle \tau_1, s_1\rangle, \ldots, \langle \tau_n, s_n\rangle, \\ & & s = f(s_1, \ldots, s_n, p_1(\mathbf{z}_1), \ldots, p_h(\mathbf{z}_h))\end{array}$$

  where
  - τ_i triples involving literals and variables in $\mathbf{x}, \mathbf{y}$
  - $\mathbf{z_i}$ are tuples of literals or variables in $\mathbf{x}$ or $\mathbf{y}$
  - $p_j(\mathbf{t}) \in [0, 1]$
  - f is a *scoring* function $f\colon ([0,1])^{n+h} \to [0,1]$
- Example:

  $\langle q(x), s\rangle \leftarrow \langle (x, \text{type}, \text{SportCar}), s_1\rangle, (x, \text{hasPrice}, y), s = s_1 \cdot \text{cheap}(y)$

  where e.g. $\text{cheap}(y) = \text{ls}(0, 10000, 12000)(y)$, has intended meaning to “retrieve all cheap sports car"

Example



$$G = \left\{ \begin{array}{ll} \langle (o1, IsAbout, snoopy), 0.8\rangle & \langle (o2, IsAbout, woodstock), 0.9\rangle \\ (snoopy, \text{type}, dog) & (woodstock, \text{type}, bird) \\ \langle (Dog, \text{sc}, SmallAnimal), 0.4\rangle & \langle (Bird, \text{sc}, SmallAnimal), 0.7\rangle \\ (SmallAnimal, \text{sc}, Animal) & \end{array} \right\}$$

Consider the query

$$\langle q(x), s\rangle \quad \leftarrow \quad \langle (x, IsAbout, y), s_1\rangle, \langle (y, \text{type}, Animal), s_2\rangle, s = s_1 \cdot s_2$$

Then

$$ans(G, q) \quad = \quad \{\langle o1, 0.32\rangle, \langle o2, 0.63\rangle\}$$

Annotation domains & RDFS

- Generalisation of fuzzy RDFS
  - a triple is annotated with a value taken from a so-called annotation domain, rather than with a value in [0,1]
  - allows to deal with several domains (such as, fuzzy, temporal, provenance) and their combination, in a uniform way
- Fuzzyness
  - $\langle (\mathit{HolidayInnHotel}, \mathit{closeTo}, \mathit{IEA17Venue}), 0.7 \rangle$
  - true to some degree
- Time
  - $\langle (\mathit{umberto}, \mathit{workedFor}, \mathit{IEI}), [1992, 2001] \rangle$
  - true during 1992–2001
- Provenance
  - $\langle (\mathit{umberto}, \mathit{knows}, \mathit{salem}), \text{http://www.straccia.info/foaf.rdf} \rangle$
  - true in `http://www.straccia.info/foaf.rdf`
- Multiple Domains:

$$\langle (\mathit{CountryXYZ}, \text{type}, \mathit{Dangerous}), \langle [1975, 1983], 0.8, 0.6 \rangle \rangle$$

$\mathit{Time} \times \mathit{Fuzzy} \times \mathit{Trust}$

- Annotation Domain: idempotent, commutative semi-ring

$$D = \langle L, \oplus, \otimes, \bot, \top \rangle$$

where $\oplus$ is $\top$-annihilating, i.e.

1. $\oplus$ is idempotent, commutative, associative;
2. $\otimes$ is commutative and associative;
3. $\bot \oplus \lambda = \lambda$, $\top \otimes \lambda = \lambda$, $\bot \otimes \lambda = \bot$, and $\top \oplus \lambda = \top$;
4. $\otimes$ is distributive over $\oplus$,
   i.e. $\lambda_1 \otimes (\lambda_2 \oplus \lambda_3) = (\lambda_1 \otimes \lambda_2) \oplus (\lambda_1 \otimes \lambda_3)$;

- Induced partial order:

$$\lambda_1 \preceq \lambda_2 \iff \lambda_1 \oplus \lambda_2 = \lambda_2$$

- Annotated triple: for $\lambda \in L$

$$\langle (s, p, o), \lambda \rangle$$

The case of Fuzzy & Annotated Description Logics

Fuzzy DLs Basics

For a degree n in L or L_n

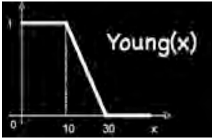
- $\langle a{:}C, n\rangle$ states that a is an instance of concept/class C with degree at least n
- $\langle C_1 \sqsubseteq C_2, n\rangle$ states that class C_1 is ausbclass of C_2 to degree n

Towards Fuzzy OWL 2 and its Profiles

- Fuzzy OWL 2 added value:
  - fuzzy concrete domains (e.g., *young*)
  - modifiers (e.g., *very* young)
  - other extensions:
    - aggregation functions: weighted sum, OWA, fuzzy integrals
    - fuzzy rough sets
    - fuzzy spatial relations
    - fuzzy numbers, ...

Fuzzy Concrete Domains

- E.g., *Small*, *Young*, *High*, *etc*. with explicit membership function
- Representation of Young Person:



$$\begin{aligned} Minor &= Person \sqcap \exists hasAge. \leq_{18} \\ YoungPerson &= Person \sqcap \exists hasAge.ls(10, 30) \end{aligned}$$

- Representation of Heavy Rain:

$$HeavyRain = Rain \sqcap \exists hasPrecipitationRate.rs(5, 7.5)$$

Fuzzy Modifiers

- *Very*, *moreOrLess*, *slightly*, etc.
- Representation of Sport Car

$$SportsCar = Car \sqcap \exists speed.very(rs(80, 250))$$

- Representation of Very Heavy Rain

$$VeryHeavyRain = Rain \sqcap \exists hasPrecipitationRate.very(rs(5, 7.5)) .$$

Aggregation Operators

- Aggregation operators: aggregate concepts, using functions such as the mean, median, weighted sum operators, etc.
- Allows to express the concept

$$0.3 \cdot \mathit{ExpensiveHotel} + 0.7 \cdot \mathit{LuxuriousHotel} \sqsubseteq \mathit{GoodHotel}$$

  - a good hotel is the weighted sum of being an expensive and luxurious hotel
- Aggregated concepts are popular in robotics:
  - to recognise complex objects from atomic ones

Fuzzy DLs Query Answering

- Conjunctive query: similar to fuzzy RDFS CQs:

$$\langle q(\mathbf{x}), s\rangle \quad \leftarrow \quad \exists \mathbf{y}.\langle \tau_1, s_1\rangle, \ldots, \langle \tau_n, s_n\rangle, \\ s = f(s_1, \ldots, s_n, p_1(\mathbf{z}_1), \ldots, p_h(\mathbf{z}_h))$$

  where

  - $\tau_1, \ldots, \tau_n$ are expressions $A(z)$ or $R(z, z')$, where A is a concept name, R is a role name, z, z' are individuals or variables in $\mathbf{x}$ or $\mathbf{y}$

- Example:

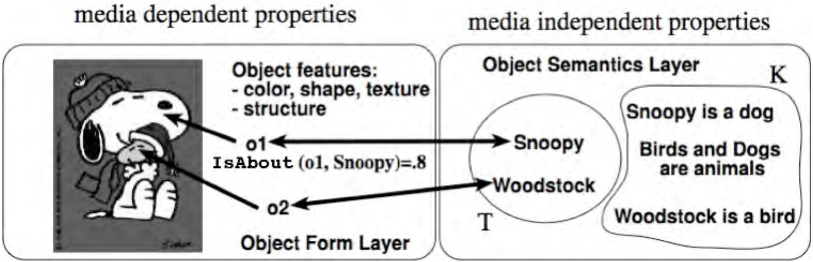
  $$\langle q(x), s\rangle \leftarrow \langle \text{SportCar}(x), s_1\rangle, \text{hasPrice}(x, y), s = s_1 \cdot \text{cheap}(y)$$

  where e.g. $\text{cheap}(y) = \text{ls}(10000, 12000)(y)$, has intended meaning to retrieve all cheap sports car.

Some Applications

- (Multimedia) Information retrieval
- Recommendation systems
- Image interpretation
- Ambient intelligence
- Ontology merging
- Matchmaking
- Decision making
- Summarization
- Robotics perception
- Software design
- Machine learning

Example



$$G = \left\{ \begin{array}{ll} \langle (o1, snoopy){:}IsAbout, 0.8\rangle & \langle (o2, woodstock){:}IsAbout, 0.9\rangle \\ snoopy{:}Dog & woodstock{:}Bird \\ \langle Dog \sqsubseteq SmallAnimal, 0.4\rangle & \langle Bird \sqsubseteq SmallAnimal, 0.7\rangle \\ SmallAnimal \sqsubseteq Animal & \end{array} \right\}$$

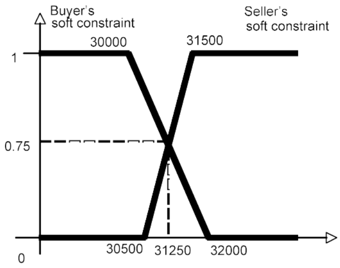
Consider the query

$$\langle q(x), s\rangle \quad \leftarrow \quad \langle IsAbout(x, y), s_1\rangle, \langle Animal(y), s_2\rangle, s = s_1 \cdot s_2$$

Then

$$ans(G, q) \quad = \quad \{\langle o1, 0.32\rangle, \langle o2, 0.63\rangle\}, \quad ans_1(G, q) = \{\langle o2, 0.63\rangle\}$$

Example (Simplified Matchmaking)



- A car seller sells an Audi TT for 31500 €, as from the catalog price.
- A buyer is looking for a sports-car, but wants to to pay not more than around 30000 €
- Classical sets: the problem relies on the crisp conditions on price
- More fine grained approach: to consider prices as fuzzy sets (as usual in negotiation)
  - Seller may consider optimal to sell above 31500 €, but can go down to 30500 €
  - The buyer prefers to spend less than 30000 €, but can go up to 32000 €

$$\begin{array}{lcl} \mathit{AudiTT} & = & \mathit{SportsCar} \sqcap \exists \mathit{hasPrice}.\mathit{rs}(30500, 31500) \\ \mathit{Query} & = & \mathit{SportsCar} \sqcap \exists \mathit{hasPrice}.\mathit{ls}(30000, 32000) \end{array}$$

  - Highest degree to which the concept $C = \mathit{AudiTT} \sqcap \mathit{Query}$ is satisfiable is 0.75 (the degree to which the Audi TT and the query matches is 0.75)
  - The car may be sold at 31250 €

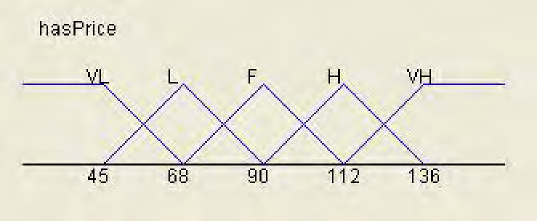
Example: Learning fuzzy GCIs from OWL data

- Learning of fuzzy GCIs from crisp OWL data
- Use Case: What are Good hotels, using TripAdvisor data?
  - Given
    - OWL 2 Ontology about meaningful city entities and their descriptions
    - TripAdvisor data about hotels and user judgments
  - We have learnt that in e.g., Pisa, Italy

$$\langle \exists hasAmenity.Babysitting \sqcap \exists hasPrice.fair \sqsubseteq Good\_Hotel, 0.782 \rangle$$

"A hotel having babysitting as amenity and a fair price is a good hotel (to degree 0.782)"

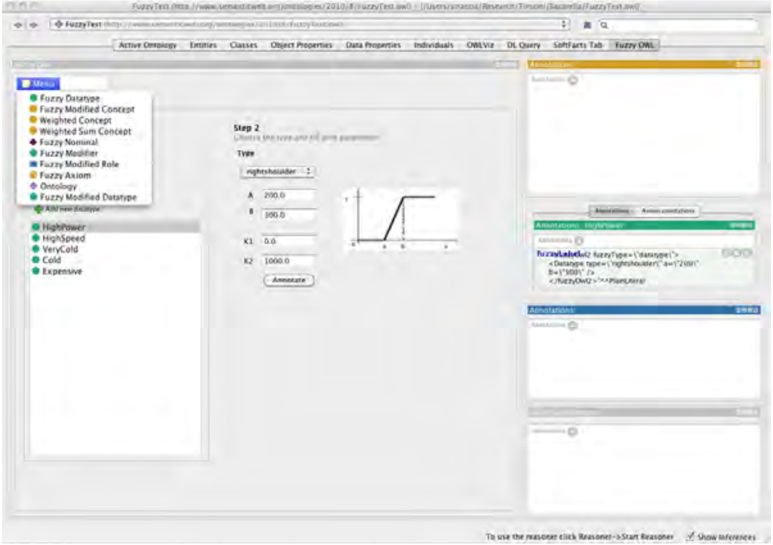
  - Real valued price attribute *hasPrice* has been automatically fuzzyfied



Representing Fuzzy OWL Ontologies in OWL

- OWL 2 is W3C standard, with classical logic semantics
  - Hence, cannot support natively Fuzzy Logic
- However, Fuzzy OWL 2, has been defined using OWL 2
  - Uses the axiom annotation feature of OWL 2
- Any Fuzzy OWL 2 ontology is a legal OWL 2 ontology

- A java parser for Fuzzy OWL 2 exists
- Protégé plug-in exists to encode Fuzzy OWL ontologies



Annotation domains & OWL

- For OWL 2, it it is like for RDFS, but annotation domain has to be a complete lattice
- Exception for OWL profiles OWL EL, OWL QL and OWL RL: annotation domains may be as for RDFS

The case of Fuzzy & Annotated Logic Programs

Fuzzy LPs Basics

- Truth space is $[0, 1]$ or $\{0, \frac{1}{n}, \ldots, \frac{n-2}{n-1}, \ldots, 1\} \quad (n \geq 1)$
- Generalized LP rules are of the form

$$\langle R(\mathbf{x}), s\rangle \quad \leftarrow \quad \exists \mathbf{y}.\langle R_1(\mathbf{z}_1), s_1\rangle, \ldots, \langle R_k(\mathbf{z}_k), s_k\rangle, \\ s = f(s_1, ..., s_k, p_1(\mathbf{z}'_1), \ldots, p_h(\mathbf{z}'_h))$$

  - Meaning of rules: "take the truth-values of all $R_i(\mathbf{z}_i), p_j(\mathbf{z}'_j)$, combine them using the truth combination function f, and assign the result to $R(\mathbf{x})$"
- Facts: ground expressions of the form $\langle R(\mathbf{c}), n\rangle$
  - Meaning of facts: "the degree of truth of $R(\mathbf{c})$ is at least n"
- Fuzzy LP: a set of facts (extensional database) and a set of rules (intentional database). No extensional relation may occur in the head of a rule

Example: Soft shopping agent

- User preferences:

$$\langle Pref_1(x,p), s\rangle \leftarrow HasPrice(x,p), s = ls(10000, 14000)(p)$$
$$\langle Pref_2(x), s\rangle \leftarrow HasKM(x,k), s = ls(13000, 17000)(k)$$
$$\langle Buy(x,p), s\rangle \leftarrow \langle Pref_1(x,p), s_p\rangle, \langle Pref_2(x_k), s_k\rangle, s = 0.7 \cdot s_p + 0.3 \cdot s_k$$

ID	MODEL	PRICE	KM
455	MAZDA 3	12500	10000
34	ALFA 156	12000	15000
1812	FORD FOCUS	11000	16000
⋮	⋮	⋮	⋮

- Problem: All tuples of the database have a score:
  - We cannot compute the score of all tuples, then rank them. Brute force approach not feasible for very large databases
- Top-k fuzzy LP problem: Determine efficiently just the top-k ranked tuples, without evaluating the score of all tuples. E.g. top-3 tuples

ID	PRICE	SCORE
1812	11000	0.6
455	12500	0.56
34	12000	0.50

Rule Languages and Semantic Web

- There are quite many LP/ASP systems (monotone/non-monotone)
  - each with its own feature set
  - some with SW interface
    - SWIProlog, DLV, ...
- More SW related: various frameworks exist ...
  - SWRL: rules with concept and role expressions as atoms
  - Datalog$^{\pm}$: Datalog with existential restriction on rule head
  - RuleML: quite large range of features
- The development of fuzzy LPs is essentially in parallel with that of classical LPs (since early '80s)
- A common problem with LP frameworks (incl. fuzzy)
  - Lack of standardised language and semantics
  - SWRL, RuleML are exceptions

Annotation domains & Datalog

- For Datalog, it it is like for RDFS
- The reasoning decision problems' complexity is inherited from their fuzzy variants. Decidable if lattice and truth space are finite, else undecidable in general

Conclusions

Conclusions & Future work

- We've overviewed basic concepts related to Fuzzyness in Semantic Web Languages, such as
  - RDFS, OWL 2, Datalog
- Semantic Web Applications:
  - Robotics, Ontology Mappings, Multimedia Object annotation, Matchmaking, (Multimedia/Distributed) Information Retrieval, Recommender Systems, User Profiling, . . .

Summary within Fuzzy Semantic Web Framework (IMHO)

Language	Mature Systems	Inference Algorithms	Query Answering
RDFS			
OWL 2			
OWL QL			
OWL EL			
OWL RL			
Rule Languages			

Emerging Field for SWLs: Enhanced Fuzzy Queries

- Fuzzy Quantified queries may provide many opportunities to improve CQ query features for any SWL: e.g.
  - *Visualise roads in which many of the recent car incidents involved severely injured people*
  - Fuzzy quantified query schema:

    $$Q \text{ of } B\,X \text{ are } A$$

  - Q is a fuzzy quantifier, e.g. *many*
  - $B\,X$ is a reference fuzzy set over which Q quantifies, e.g. *recent* (B) *car incidents* (X)
  - A is a fuzzy set imposing a condition to be satisfied, e.g. *severely injured people*
- Fuzzy Queries may be applied both to crisp ontologies as well as to fuzzy ontologies

That’s it !

References

[1] Daniel J. Abadi, Adam Marcus, Samuel Madden, and Kate Hollenbach. SW-Store: a vertically partitioned DBMS for semantic web data management. *VLDB Journal*, 18(2):385–406, 2009.

[2] Sudhir Agarwal and Steffen Lamparter. Smart: A semantic matchmaking portal for electronic markets. In *CEC '05: Proceedings of the Seventh IEEE International Conference on E-Commerce Technology (CEC'05)*, pages 405–408, Washington, DC, USA, 2005. IEEE Computer Society.

[3] Troels Andreasen and Henrik Bulskov. Conceptual querying through ontologies. *Fuzzy Sets Syst.*, 160(15):2159–2172, August 2009.

[4] Franz Baader, Diego Calvanese, Deborah McGuinness, Daniele Nardi, and Peter F. Patel-Schneider, editors. *The Description Logic Handbook: Theory, Implementation, and Applications*. Cambridge University Press, 2003.

[5] Franz Baader and Rafael Peñaloza. Are fuzzy description logics with general concept inclusion axioms decidable? In *Proceedings of 2011 IEEE International Conference on Fuzzy Systems (Fuzz-IEEE 2011)*. IEEE Press, 2011.

[6] Franz Baader and Rafael Peñaloza. GCIs make reasoning in fuzzy DLs with the product t-norm undecidable. In *Proceedings of the 24th International Workshop on Description Logics (DL-11)*. CEUR Electronic Workshop Proceedings, 2011.

[7] Radu Balaj and Adrian Groza. Detecting influenza epidemics based on real-time semantic analysis of Twitter streams. In *Proceedings of the 3rd International Conference on Modelling and Development of Intelligent Systems (MDIS 2013)*, pages 30–39, 2013.

[8] J. F. Baldwin, T. P. Martin, and B. W. Pilsworth. *Fril - Fuzzy and Evidential Reasoning in Artificial Intelligence*. Research Studies Press Ltd, 1995.

[9] J. F. Baldwin, T. P. Martin, and B. W. Pilsworth. Applications of fuzzy computation: Knowledge based systems: Knowledge representation. In E. H. Ruspini, P. Bonnissone, and W. Pedrycz, editors, *Handbook of Fuzzy Computing*. IOP Publishing, 1998.

[10] James C. Bezdek. *Pattern Recognition with Fuzzy Objective Function Algorithms*. Springer Verlag, 1981.

[11] Fernando Bobillo. The role of crisp elements in fuzzy ontologies: The case of fuzzy OWL 2 EL. *IEEE Transactions on Fuzzy Systems*, 24:1193–1209, 2016.

[12] Fernando Bobillo, Félix Bou, and Umberto Straccia. On the failure of the finite model property in some fuzzy description logics. *Fuzzy Sets and Systems*, 172(1):1–12, 2011.

[13] Fernando Bobillo, Marco Cerami, Francesc Esteva, Àngel García-Cerdaña, Rafael Peñaloza, and Umberto Straccia. Fuzzy description logics in the framework of mathematical fuzzy logic. In Carles Noguera Petr Cintula, Christian Fermüller, editor, *Handbook of Mathematical Fuzzy Logic, Volume 3*, volume 58 of *Studies in Logic, Mathematical Logic and Foundations*, chapter 16, pages 1105–1181. College Publications, 2015.

[14] Fernando Bobillo, Miguel Delgado, and Juan Gómez-Romero. A crisp representation for fuzzy $\mathcal{SHOIN}$ with fuzzy nominals and general concept inclusions. In *Proceedings of the 2nd Workshop on Uncertainty Reasoning for the Semantic Web (URSW-06)*, November 2006.

[15] Fernando Bobillo, Miguel Delgado, and Juan Gómez-Romero. A crisp representation for fuzzy $\mathcal{SHOIN}$ with fuzzy nominals and general concept inclusions. In *Uncertainty Reasoning for the Semantic Web I*, volume 5327 of *Lecture Notes in Computer Science*, pages 174–188. Springer Verlag, 2008.

[16] Fernando Bobillo, Miguel Delgado, and Juan Gómez-Romero. Delorean: A reasoner for fuzzy OWL 1.1. In *Proceedings of the 4th International Workshop on Uncertainty Reasoning for the Semantic Web (URSW 2008)*, volume 423. CEUR Workshop Proceedings, 10 2008.

[17] Fernando Bobillo, Miguel Delgado, and Juan Gómez-Romero. Optimizing the crisp representation of the fuzzy description logic $\mathcal{SROIQ}$. In *Uncertainty Reasoning for the Semantic Web I*, volume 5327 of *Lecture Notes in Computer Science*, pages 189–206. Springer Verlag, 2008.

[18] Fernando Bobillo, Miguel Delgado, Juan Gómez-Romero, and Umberto Straccia. Fuzzy description logics under Gödel semantics. *International Journal of Approximate Reasoning*, 50(3):494–514, 2009.

[19] Fernando Bobillo, Miguel Delgado, Juan Gómez-Romero, and Umberto Straccia. Joining Gödel and Zadeh fuzzy logics in fuzzy description logics. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 20:475–508, 2012.

[20] Fernando Bobillo and Umberto Straccia. A fuzzy description logic with product t-norm. In *Proceedings of the IEEE International Conference on Fuzzy Systems (Fuzz-IEEE-07)*, pages 652–657. IEEE Computer Society, 2007.

[21] Fernando Bobillo and Umberto Straccia. fuzzyDL: An expressive fuzzy description logic reasoner. In *2008 International Conference on Fuzzy Systems (FUZZ-08)*, pages 923–930. IEEE Computer Society, 2008.

[22] Fernando Bobillo and Umberto Straccia. On qualified cardinality restrictions in fuzzy description logics under Łukasiewicz semantics. In Luis Magdalena, Manuel Ojeda-Aciego, and José Luis Verdegay, editors, *Proceedings of the 12th International Conference of Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU 2008)*, pages 1008–1015, June 2008.

[23] Fernando Bobillo and Umberto Straccia. Towards a crisp representation of fuzzy description logics under Łukasiewicz semantics. In Aijun An, Stan Matwin, Zbigniew W. Raś, and Dominik Ślęzak, editors, *Proceedings of the 17th International Symposium on Methodologies for Intelligent Systems (ISMIS 2008)*, volume 4994 of *Lecture Notes in Computer Science*, pages 309–318. Springer Verlag, May 2008.

[24] Fernando Bobillo and Umberto Straccia. Extending datatype restrictions in fuzzy description logics. In *Proceedings of the 9th International Conference on Intelligent Systems Design and Applications (ISDA-09)*, pages 785–790. IEEE Computer Society, 2009.

[25] Fernando Bobillo and Umberto Straccia. Fuzzy description logics with fuzzy truth values. In João P. B. Carvalho, Didier Dubois, Uzay Kaymak, and João M. C. Sousa, editors, *Proceedings of the 13th World Congress of the International Fuzzy Systems Association and 6th Conference of the European Society for Fuzzy Logic and Technology (IFSA-EUSFLAT 2009)*, pages 189–194, July 2009.

[26] Fernando Bobillo and Umberto Straccia. Fuzzy description logics with general t-norms and datatypes. *Fuzzy Sets and Systems*, 160(23):3382—3402, 2009.

[27] Fernando Bobillo and Umberto Straccia. An OWL ontology for fuzzy OWL 2. In *Proceedings of the 18th International Symposium on Methodologies for Intelligent Systems (ISMIS-09)*, volume 5722 of *Lecture Notes in Computer Science*, pages 151–160. Springer-Verlag, September 2009.

[28] Fernando Bobillo and Umberto Straccia. Supporting fuzzy rough sets in fuzzy description logics. In *Proceedings of the 10th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU-09)*, volume 5590 of *Lecture Notes in Artificial Intelligence*, pages 676–687. Springer Verlag, 2009.

[29] Fernando Bobillo and Umberto Straccia. Finite fuzzy description logics: A crisp representation for finite fuzzy $\mathcal{ALCH}$. In Fernando Bobillo, Rommel Carvalho, Paulo C. G. da Costa, Claudia d'Amato, Nicola Fanizzi, Kathryn B. Laskey, Kenneth J. Laskey, Thomas Lukasiewicz, Trevor Martin, Matthias Nickles, and Michael Pool, editors, *Proceedings of the 6th ISWC Workshop on Uncertainty Reasoning for the Semantic Web (URSW 2010)*, volume 654, pages 61–72. CEUR Workshop Proceedings, November 2010.

[30] Fernando Bobillo and Umberto Straccia. Representing fuzzy ontologies in owl 2. In *Proceedings of the 19th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2010)*, pages 2695–2700. IEEE Press, July 2010.

[31] Fernando Bobillo and Umberto Straccia. Aggregation operators and fuzzy OWL 2. In *Proceedings of the 20th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2011)*, pages 1727–1734. IEEE Press, June 2011.

[32] Fernando Bobillo and Umberto Straccia. Fuzzy ontologies and fuzzy integrals. In *Proceedings of the 11th International Conference on Intelligent Systems Design and Applications (ISDA 2011)*, pages 1311–1316. IEEE Press, November 2011.

[33] Fernando Bobillo and Umberto Straccia. Fuzzy ontology representation using OWL 2. *International Journal of Approximate Reasoning*, 52:1073–1094, 2011.

[34] Fernando Bobillo and Umberto Straccia. Reasoning with the finitely many-valued Łukasiewicz fuzzy description logic $\mathcal{SROIQ}$. *Information Sciences*, 181:758–778, 2011.

[35] Fernando Bobillo and Umberto Straccia. Generalized fuzzy rough description logics. *Information Sciences*, 189:43–62, 2012.

[36] Fernando Bobillo and Umberto Straccia. Aggregation operators for fuzzy ontologies. *Applied Soft Computing*, 13(9):3816–3830, 2013.

[37] Fernando Bobillo and Umberto Straccia. Finite fuzzy description logics and crisp representations. In Fernando Bobillo, Paulo C. G. da Costa, Claudia d'Amato, Nicola Fanizzi, Kathryn Laskey, Ken Laskey, Thomas Lukasiewicz, Matthias Nickles, and Michael Pool, editors, *Uncertainty Reasoning for the Semantic Web II*, volume 7123 of *Lecture Notes in Computer Science*, pages 102–121. Springer Verlag, 2013.

[38] Fernando Bobillo and Umberto Straccia. General concept inclusion absorptions for fuzzy description logics: A first step. In *Proceedings of the 26th International Workshop on Description Logics (DL-13)*, volume 1014 of *CEUR Workshop Proceedings*, pages 513–525. CEUR-WS.org, 2013.

[39] Fernando Bobillo and Umberto Straccia. A MILP-based decision procedure for the (fuzzy) description logic $\mathcal{ALCB}$. In *Proceedings of the 27th International Workshop on Description Logics (DL 2014)*, volume 1193, pages 378–390. CEUR Workshop Proceedings, July 2014.

[40] Fernando Bobillo and Umberto Straccia. On partitioning-based optimisations in expressive fuzzy description logics. In *Proceedings of the 24th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE-15)*. IEEE Press, August 2015.

[41] Fernando Bobillo and Umberto Straccia. The fuzzy ontology reasoner *fuzzyDL*. *Knowledge-Based Systems*, 95:12 – 34, 2016.

[42] Fernando Bobillo and Umberto Straccia. Optimising fuzzy description logic reasoners with general concept inclusions absorption. *Fuzzy Sets and Systems*, 292:98–129, 2016.

[43] Fernando Bobillo and Umberto Straccia. Generalizing type-2 fuzzy ontologies and type-2 fuzzy description logics. *International Journal of Approximate Reasoning*, 87:40 –66, 2017.

[44] Fernando Bobillo and Umberto Straccia. Reasoning within fuzzy owl 2 el revisited. *Fuzzy Sets and Systems*, 351:1–40, 2018.

[45] P. Bonatti and A. Tettamanzi. Some complexity results on fuzzy description logics. In A. Petrosino V. Di Gesù, F. Masulli, editor, *WILF 2003 International Workshop on Fuzzy Logic and Applications*, LNCS 2955, Berlin, 2004. Springer Verlag.

[46] Stefan Borgwardt, Marco Cerami, and Rafael Peñaloza. Subsumption in finitely valued fuzzy $\mathcal{EL}$. LTCS-Report 15-06, Chair for Automata Theory, Institute for Theoretical Computer Science, Technische Universität Dresden, Dresden, Germany, 2015. See http://lat.inf.tu-dresden.de/research/reports.html.

[47] Stefan Borgwardt, Marco Cerami, and Rafael Peñaloza. The complexity of subsumption in fuzzy EL. In *Proceedings of the Twenty-Fourth International Joint Conference on Artificial Intelligence, IJCAI 2015, Buenos Aires, Argentina, July 25-31, 2015*, pages 2812–2818. IJCAI/AAAI, 2015.

[48] Stefan Borgwardt, Marco Cerami, and Rafael Peñaloza. The complexity of fuzzy $\mathcal{EL}$ under the łukasiewicz t-norm. *International Journal of Approximate Reasoning*, 91:179–201, 2017.

[49] Stefan Borgwardt, Felix Distel, and Rafael Peñaloza. The limits of decidability in fuzzy description logics with general concept inclusions. *Artificial Intelligence*, 218:23–55, 2015.

[50] Stefan Borgwardt, Felix Distel, and Rafael Peñaloza. How fuzzy is my fuzzy description logic? In *Proceedings of the 6th International Joint Conference on Automated Reasoning (IJCAR-12)*, volume 7364 of *Lecture Notes in Artificial Intelligence*, pages 82–96, Manchester, UK, 2012. Springer-Verlag.

[51] Stefan Borgwardt, Felix Distel, and Rafael Peñaloza. Non-Gödel negation makes unwitnessed consistency undecidable. In *Proceedings of the 2012 International Workshop on Description Logics (DL-2012)*, volume 846. CEUR-WS.org, 2012.

[52] Stefan Borgwardt, Theofilos P. Mailis, Rafael Peñaloza, and Anni-Yasmin Turhan. Answering fuzzy conjunctive queries over finitely valued fuzzy ontologies. *Journal of Data Semantics*, 5(2):55–75, 2016.

[53] Stefan Borgwardt and Rafael Peñaloza. A tableau algorithm for $\mathcal{SROIQ}$ under infinitely valued Gödel semantics. LTCS-Report 15-18, Chair for Automata Theory, Technische Universität Dresden, Germany, 2015.

[54] Stefan Borgwardt and Rafael Peñaloza. Description logics over lattices with multi-valued ontologies. In *Proceedings of the Twenty-Second International Joint Conference on Artificial Intelligence (IJCAI-11)*, pages 768–773, 2011.

[55] Stefan Borgwardt and Rafael Peñaloza. Finite lattices do not make reasoning in $\mathcal{ALCI}$ harder. In *Proceedings of the 7th International Workshop on Uncertainty Reasoning for the Semantic Web (URSW-11)*, volume 778, pages 51–62. CEUR-WS.org, 2011.

[56] Stefan Borgwardt and Rafael Peñaloza. Fuzzy ontologies over lattices with t-norms. In *Proceedings of the 24th International Workshop on Description Logics (DL-11)*. CEUR Electronic Workshop Proceedings, 2011.

[57] Stefan Borgwardt and Rafael Peñaloza. A tableau algorithm for fuzzy description logics over residuated De Morgan lattices. In Markus Krötzsch and Umberto Straccia, editors, *Proceedings of the 6th International Conference on Web Reasoning and Rule Systems (RR-12)*, volume 7497 of *Lecture Notes in Computer Science*, pages 9–24. Springer, 2012.

[58] Stefan Borgwardt and Rafael Peñaloza. Undecidability of fuzzy description logics. In *Proceedings of the 13th International Conference on Principles of Knowledge Representation and Reasoning (KR-12)*, pages 232–242, Rome, Italy, 2012. AAAI Press.

[59] Stefan Borgwardt and Rafael Peñaloza. About subsumption in fuzzy EL. In *Informal Proceedings of the 26th International Workshop on Description Logics (DL-13)*, CEUR-WS.org, pages 526–538, 2013.

[60] Stefan Borgwardt and Rafael Peñaloza. The complexity of lattice-based fuzzy description logics. *Journal of Data Semantics*, 2(1):1–19, 2013.

[61] Stefan Borgwardt and Rafael Peñaloza. Positive subsumption in fuzzy EL with general t-norms. In *IJCAI 2013, Proceedings of the 23rd International Joint Conference on Artificial Intelligence, Beijing, China, August 3-9, 2013*. IJCAI/AAAI, 2013.

[62] Stefan Borgwardt and Rafael Peñaloza. Algorithms for reasoning in very expressive description logics under infinitely valued gödel semantics. *International Journal of Approximate Reasoning*, 83:60–101, 2017.

[63] Félix Bou, Marco Cerami, and Francesc Esteva. Finite-valued Łukasiewicz modal logic is Pspace-complete. In *Proceedings of the 22nd International Joint Conference on Artificial Intelligence (IJCAI-11)*, pages 774–779, 2011.

[64] Dan Brickley and R.V. Guha. RDF Vocabulary Description Language 1.0: RDF Schema. W3C Recommendation, W3C, 2004. `http://www.w3.org/TR/rdf-schema/`.

[65] F. Bueno, D. Cabeza, M. Carro, M. Hermenegildo, P. López-García, and G. Puebla. The Ciao prolog system. Reference manual. Technical Report CLIPS3/97.1, School of Computer Science, Technical University of Madrid (UPM), 1997. Available at http://www.cliplab.org/Software/Ciao/.

[66] Peter Buneman and Egor Kostylev. Annotation algebras for rdfs. In *The Second International Workshop on the role of Semantic Web in Provenance Management (SWPM-10)*. CEUR Workshop Proceedings, 2010.

[67] Silvia Calegari and Elie Sanchez. Object-fuzzy concept network: An enrichment of ontologies in semantic information retrieval. *Journal of the American Society for Information Science and Technology*, 59(13):2171–2185, November 2008.

[68] Andrea Calì, Georg Gottlob, and Thomas Lukasiewicz. Datalog±: A unified approach to ontologies and integrity constraints. In *Proceedings of the 12th International Conference on Database Theory*, pages 14–30, New York, NY, USA, 2009. ACM.

[69] J. Calmet, J. Lu, M. Rodriguez, and J. Schü. Signed formula logic programming: operational semantics and applications. In Zbigniew W. Rás and Maciek Michalewicz, editors, *Proceedings of the 9th International Symposium on Foundations of Intelligent Systems*, volume 1079 of *Lecture Notes in Artificial Intelligence*, pages 202–211, Berlin, 1996. Springer.

[70] True H. Cao. Annotated fuzzy logic programs. *Fuzzy Sets and Systems*, 113(2):277–298, 2000.

[71] Christer Carlsson, Matteo Brunelli, and József Mezei. Decision making with a fuzzy ontology. *Soft Computing*, 16(7):1143–1152, July 2012.

[72] Marco Cerami, Francesc Esteva, and Fèlix Bou. Decidability of a description logic over infinite-valued product logic. In *Proceedings of the Twelfth International Conference on Principles of Knowledge Representation and Reasoning (KR-10)*. AAAI Press, 2010.

[73] Marco Cerami and Umberto Straccia. On the undecidability of fuzzy description logics with gcis with lukasiewicz t-norm. Technical report, Computing Research Repository, 2011. Available as CoRR technical report at http://arxiv.org/abs/1107.4212.

[74] Marco Cerami and Umberto Straccia. Undecidability of KB satisfiability for Ł-$\mathcal{ALC}$ with GCIs. Unpublished Manuscript, July 2011.

[75] Rung-Ching Chen, Cho Tsan Bau, and Chun-Ju Yeh. Merging domain ontologies based on the WordNet system and fuzzy formal concept analysis techniques. *Applied Soft Computing*, 11(2):1908–1923, 2011.

[76] Alexandros Chortaras, Giorgos B. Stamou, and Andreas Stafylopatis. Adaptation of weighted fuzzy programs. In *Artificial Neural Networks - ICANN 2006, 16th International Conference, Part II*, pages 45–54, 2006.

[77] Alexandros Chortaras, Giorgos B. Stamou, and Andreas Stafylopatis. Integrated query answering with weighted fuzzy rules. In *9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU-07)*, volume 4724 of *Lecture Notes in Computer Science*, pages 767–778. Springer Verlag, 2007.

[78] Alexandros Chortaras, Giorgos B. Stamou, and Andreas Stafylopatis. Top-down computation of the semantics of weighted fuzzy logic programs. In *Web Reasoning and Rule Systems, First International Conference, (RR-07)*, pages 364–366, 2007.

[79] Simona Colucci, Tommaso Di Noia, Azzurra Ragone, Michele Ruta, Umberto Straccia, and Eufemia Tinelli. Informative top-k retrieval for advanced skill management. In *Semantic Web Information Management*, chapter 19, pages 449–476. Springer Verlag, 2010.

[80] Paulo C. G. Costa, Kathryn B. Laskey, and Thomas Lukasiewicz. Uncertainty representation and reasoning in the semantic web. In *Semantic Web Engineering in the Knowledge Society*, pages 315–340. IGI Global, 2008.

[81] B. Cuenca-Grau, I. Horrocks, B. Motik, B. Parsia, P.F. Patel-Schneider, and U. Sattler. OWL 2: The next step for OWL. *Journal of Web Semantics*, 6(4):309–322, 2008.

[82] Carlos Viegas Damásio, J. Medina, and M. Ojeda Aciego. Sorted multi-adjoint logic programs: Termination results and applications. In *Proceedings of the 9th European Conference on Logics in Artificial Intelligence (JELIA-04)*, volume 3229 of *Lecture Notes in Computer Science*, pages 252–265. Springer Verlag, 2004.

[83] Carlos Viegas Damásio, J. Medina, and M. Ojeda Aciego. A tabulation proof procedure for residuated logic programming. In *Proceedings of the 6th European Conference on Artificial Intelligence (ECAI-04)*, 2004.

[84] Carlos Viegas Damásio, J. Medina, and M. Ojeda Aciego. Termination results for sorted multi-adjoint logic programs. In *Proceedings of the 10th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-04)*, pages 1879–1886, 2004.

[85] Carlos Viegas Damásio, Jeff Z. Pan, Giorgos Stoilos, and Umberto Straccia. An approach to representing uncertainty rules in ruleml. In *Second International Conference on Rules and Rule Markup Languages for the Semantic Web (RuleML-06)*, pages 97–106. IEEE, 2006.

[86] Carlos Viegas Damasio, Jeff Z. Pan, Giorgos Stoilos, and Umberto Straccia. Representing uncertainty rules in ruleml. *Fundamenta Informaticae*, 82(3):265–288, 2008.

[87] Carlos Viegas Damásio and Luís Moniz Pereira. A survey of paraconsistent semantics for logic programs. In D. Gabbay and P. Smets, editors, *Handbook of Defeasible Reasoning and Uncertainty Management Systems*, pages 241–320. Kluwer, 1998.

[88] Carlos Viegas Damásio and Luís Moniz Pereira. Antitonic logic programs. In *Proceedings of the 6th European Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR-01)*, volume 2173 of *Lecture Notes in Computer Science*. Springer-Verlag, 2001.

[89] Carlos Viegas Damásio and Luís Moniz Pereira. Monotonic and residuated logic programs. In Salem Benferhat and Philippe Besnard, editors, *Symbolic and Quantitative Approaches to Reasoning with Uncertainty, 6th European Conference, ECSQARU 2001, Toulouse, France, September 19-21, 2001, Proceedings*, volume 2143 of *Lecture Notes in Computer Science*, pages 748–759. Springer, 2001.

[90] Carlos Viegas Damásio and Luís Moniz Pereira. Sorted monotonic logic programs and their embeddings. In *Proceedings of the 10th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-04)*, pages 807–814, 2004.

[91] C.V. Damásio, J. Medina, and M. Ojeda-Aciego. A tabulation procedure for first-order residuated logic programs. In *Proceedings of the 11th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-06)*, 2006.

[92] C.V. Damásio, J. Medina, and M. Ojeda-Aciego. A tabulation procedure for first-order residuated logic programs. In *Proceedings of the IEEE World Congress on Computational Intelligence (section Fuzzy Systems) (WCCI-06)*, pages 9576–9583, 2006.

[93] C.V. Damásio, J. Medina, and M. Ojeda-Aciego. Termination of logic programs with imperfect information: applications and query procedure. *Journal of Applied Logic*, 7(5):435–458, 2007.

[94] Mathieu d'Aquin, Jean Lieber, and Amedeo Napoli. Towards a semantic portal for oncology using a description logic with fuzzy concrete domains. In Elie Sanchez, editor, *Fuzzy Logic and the Semantic Web*, Capturing Intelligence, pages 379–393. Elsevier, 2006.

[95] Stamatia Dasiopoulou and Ioannis Kompatsiaris. Trends and issues in description logics frameworks for image interpretation. In *Artificial Intelligence: Theories, Models and Applications, 6th Hellenic Conference on AI*, volume 6040 of *Lecture Notes in Computer Science*, pages 61–70, 2010.

[96] Stamatia Dasiopoulou, Ioannis Kompatsiaris, and Michael G. Strintzis. Applying fuzzy dls in the extraction of image semantics. *Journal of Data Semantics*, 14:105–132, 2009.

[97] Stamatia Dasiopoulou, Ioannis Kompatsiaris, and Michael G. Strintzis. Investigating fuzzy dls-based reasoning in semantic image analysis. *Multimedia Tools and Applications*, 49(1):167–194, August 2010.

[98] M. Denecker, V. Marek, and M. Truszczyński. Approximations, stable operators, well-founded fixpoints and applications in nonmonotonic reasoning. In J. Minker, editor, *Logic-Based Artifical Intelligence*, pages 127–144. Kluwer Academic Publishers, 2000.

[99] M. Denecker, N. Pelov, and M. Bruynooghe. Ultimate well-founded and stable semantics for logic programs with aggregates. In Philippe Codognet, editor, *17th International Conference on Logic Programming*, volume 2237 of *Lecture Notes in Computer Science*, pages 212–226. Springer, 2001.

[100] Marc Denecker, Victor W. Marek, and Mirosław Truszczyński. Uniform semantic treatment of default and autoepistemic logics. In A.G. Cohn, F. Giunchiglia, and B. Selman, editors, *Proceedings of the 7th International Conference on Principles of Knowledge Representation and Reasoning*, pages 74–84. Morgan Kaufman, 2000.

[101] Marc Denecker, Victor W. Marek, and Mirosław Truszczyński. Ultimate approximations. Technical Report CW 320, Katholieke Iniversiteit Leuven, September 2001.

[102] Marc Denecker, Victor W. Marek, and Mirosław Truszczyński. Ultimate approximations in nonmonotonic knowledge representation systems. In D. Fensel, F. Giunchiglia, D. McGuinness, and M. Williams, editors, *Principles of Knowledge Representation and Reasoning: Proceedings of the 8th International Conference*, pages 177–188. Morgan Kaufmann, 2002.

[103] Natalia Díaz-Rodríguez, Olmo León-Cadahía, Manuel Pegalajar-Cuéllar, Johan Lilius, and Miguel Delgado. Handling real-world context-awareness, uncertainty and vagueness in real-time human activity tracking and recognition with a fuzzy ontology-based hybrid method. *Sensors*, 14(10):18131–18171, 2014.

[104] Natalia Díaz-Rodríguez, Manuel Pegalajar-Cuéllar, Johan Lilius, and Miguel Delgado. A fuzzy ontology for semantic modelling and recognition of human behaviour. *Knowledge-Based Systems*, 66:46–60, 2014.

[105] Dzung Dinh-Khac, Steffen Hölldobler, and Dinh-Khang Tran. The fuzzy linguistic description logic $\mathcal{ALC}_{FL}$. In *Proceedings of the 11th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-06)*, pages 2096–2103. E.D.K., Paris, 2006.

[106] Didier Dubois, Jerome Mengin, and Henri Prade. Possibilistic uncertainty and fuzzy features in description logic. A preliminary discussion. In Elie Sanchez, editor, *Capturing Intelligence: Fuzzy Logic and the Semantic Web*. Elsevier, 2006.

[107] Didier Dubois and Henri Prade. *Fuzzy Sets and Systems*. Academic Press, 1980.

[108] Didier Dubois and Henri Prade. Can we enforce full compositionality in uncertainty calculi? In *Proceedings of the 12th National Conference on Artificial Intelligence (AAAI-94)*, pages 149–154, Seattle, Washington, 1994.

[109] Didier Dubois and Henri Prade. Possibility theory, probability theory and multiple-valued logics: A clarification. *Annals of Mathematics and Artificial Intelligence*, 32(1-4):35–66, 2001.

[110] Rafee Ebrahim. Fuzzy logic programming. *Fuzzy Sets and Systems*, 117(2):215–230, 2001.

[111] Markus Eich, Ronny Hartanto, Sebastian Kasperski, Sankaranarayanan Natarajan, and Johannes Wollenberg. Towards coordinated multirobot missions for lunar sample collection in an unknown environment. *Journal of Field Robotics*, 31(1):35–74, 2014.

[112] Thomas Eich. An application of fuzzy dl-based semantic perception to soil container classification. In *IEEE International Conference on Technologies for Practical Robot Applications (TePRA-13)*, pages 1–6. IEEE Press, 2013.

[113] Carles Fernández. *Understanding image sequences: the role of ontologies in cognitive vision systems*. PhD thesis, Universitat Autònoma de Barcelona, Spain, 2010.

[114] Umberto Straccia Fernando Bobillo. Reducing the size of the optimization problems in fuzzy ontology reasoning. In *Proceedings of the 11th International Workshop on Uncertainty Reasoning for the Semantic Web (URSW-15)*, volume 1479 of *CEUR Workshop Proceedings*, pages 54–59. CEUR-WS.org, 2015.

[115] Fire. `http://www.image.ece.ntua.gr/~nsimou/FiRE/`.

[116] M. C. Fitting. The family of stable models. *Journal of Logic Programming*, 17:197–225, 1993.

[117] M. C. Fitting. Fixpoint semantics for logic programming - a survey. *Theoretical Computer Science*, 21(3):25–51, 2002.

[118] Melvin Fitting. A Kripke-Kleene-semantics for general logic programs. *Journal of Logic Programming*, 2:295–312, 1985.

[119] Melvin Fitting. Pseudo-Boolean valued Prolog. *Studia Logica*, XLVII(2):85–91, 1987.

[120] Melvin Fitting. Bilattices and the semantics of logic programming. *Journal of Logic Programming*, 11:91–116, 1991.

[121] Fuzzy OWL 2 Web Ontology Language . *http://www.straccia.info/software/FuzzyOWL/*. ISTI - CNR, 2011.

[122] Mingxia Gao and Chunnian Liu. Extending OWL by fuzzy description logic. In *Proceedings of the 17th IEEE International Conference on Tools with Artificial Intelligence (ICTAI-05)*, pages 562–567, Washington, DC, USA, 2005. IEEE Computer Society.

[123] Dusan Guller. Procedural semantics for fuzzy disjunctive programs. In Matthias Baaz and Andrei Voronkov, editors, *Logic for Programming, Artificial Intelligence, and Reasoning 9th International Conference, LPAR 2002, Tbilisi, Georgia, October 14-18, 2002, Proceedings*, volume 2514 of *Lecture Notes in Computer Science*, pages 247–261. Springer, 2002.

[124] Dusan Guller. Semantics for fuzzy disjunctive programs with weak similarity. In Ajith Abraham and Mario Köppen, editors, *Hybrid Information Systems, First International Workshop on Hybrid Intelligent Systems, Adelaide, Australia, December 11-12, 2001, Proceedings*, Advances in Soft Computing, pages 285–299. Physica-Verlag, 2002.

[125] Volker Haarslev, Hsueh-Ieng Pai, and Nematollaah Shiri. Optimizing tableau reasoning in alc extended with uncertainty. In *Proceedings of the 2007 International Workshop on Description Logics (DL-07)*, 2007.

[126] Hashim Habiballa. Resolution strategies for fuzzy description logic. In *Proceedings of the 5th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT-07)*, volume 2, pages 27–36, 2007.

[127] Reiner Hähnle. Uniform notation of tableaux rules for multiple-valued logics. In *Proceedings of the International Symposium on Multiple-Valued Logic*, pages 238–245. IEEE Press, Los Alamitos, 1991.

[128] Reiner Hähnle. Advanced many-valued logics. In Dov M. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic, 2nd Edition*, volume 2. Kluwer, Dordrecht, Holland, 2001.

[129] Petr Hájek. *Metamathematics of Fuzzy Logic*. Kluwer, 1998.

[130] Petr Hájek. Making fuzzy description logics more general. *Fuzzy Sets and Systems*, 154(1):1–15, 2005.

[131] Petr Hájek. What does mathematical fuzzy logic offer to description logic? In Elie Sanchez, editor, *Fuzzy Logic and the Semantic Web*, Capturing Intelligence, chapter 5, pages 91–100. Elsevier, 2006.

[132] C.J. Hinde. Fuzzy prolog. *International Journal Man.-Machine Studies*, 24:569–595, 1986.

[133] Steffen Hölldobler, Tran Dinh Khang, and Hans-Peter Störr. A fuzzy description logic with hedges as concept modifiers. In Nguyen Hoang Phuong, Hung T. Nguyen, Nguyen Cat Ho, and Pratit Santiprabhob, editors, *Proceedings InTech/VJFuzzy'2002*, pages 25–34, Hanoi, Vietnam, 2002. Institute of Information Technology, Vietnam Center for Natural Science and Technology, Science and Technics Publishing House, Hanoi, Vietnam.

[134] Steffen Hölldobler, Nguyen Hoang Nga, and Tran Dinh Khang. The fuzzy description logic $\mathcal{ALC}_{FH}$. In *Proceeedings of the International Workshop on Description Logics (DL-05)*, 2005.

[135] Steffen Hölldobler, Hans-Peter Störr, and Tran Dinh Khang. The fuzzy description logic $\mathcal{ALC}_{FH}$ with hedge algebras as concept modifiers. *Journal of Advanced Computational Intelligence and Intelligent Informatics (JACIII)*, 7(3):294 – 305, 2003.

[136] Steffen Hölldobler, Hans-Peter Störr, and Tran Dinh Khang. A fuzzy description logic with hedges and concept modifiers. In *Proceedings of the 10th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-04)*, 2004.

[137] Steffen Hölldobler, Hans-Peter Störr, and Tran Dinh Khang. The subsumption problem of the fuzzy description logic $\mathcal{ALC}_{FH}$. In *Proceedings of the 10th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-04)*, 2004.

[138] http://ruleml.org/index.html. The rule markup initiative.

[139] Giovambattista Ianni, Thomas Krennwallner, Alessandra Martello, and Axel Polleres. A rule system for querying persistent RDFS data. In *The Semantic Web: Research and Applications, 6th European Semantic Web Conference (ESWC-2009)*, pages 857–862, 2009.

[140] Josué Iglesias and Jens Lehmann. Towards integrating fuzzy logic capabilities into an ontology-based inductive logic programming framework. In *Proceedings of the 11th International Conference on Intelligent Systems Design and Applications (ISDA 2011)*, pages 1323–1328, 2011.

[141] Mitsuru Ishizuka and Naoki Kanai. Prolog-ELF: incorporating fuzzy logic. In *Proceedings of the 9th International Joint Conference on Artificial Intelligence (IJCAI-85)*, pages 701–703, Los Angeles, CA, 1985.

[142] Yuncheng Jiang, Hai Liu, Yong Tang, and Qimai Chen. Semantic decision making using ontology-based soft sets. *Mathematical and Computer Modelling*, 53(5–6):1140–1149, 2011.

[143] Yuncheng Jiang, Yong Tang, Qimai Chen, Ju Wang, and Suqin Tang. Extending soft sets with description logics. *Computers & Mathematics with Applications*, 59(6):2087–2096, 2010.

[144] Yuncheng Jiang, Yong Tang, Ju Wang, Peimin Deng, and Suqin Tang. Expressive fuzzy description logics over lattices. *Knowledge-Based Systems*, 23:150–161, March 2010.

[145] Yuncheng Jiang, Yong Tang, Ju Wang, and Suqin Tang. Reasoning within intuitionistic fuzzy rough description logics. *Information Sciences*, 179(2362–2378), 2009.

[146] Yuncheng Jiang, Yong Tang, Ju Wang, and Suqin Tang. Representation and reasoning of context-dependant knowledge in distributed fuzzy ontologies. *Expert Systems with Applications*, 37(8):6052–6060, 2010.

[147] Yuncheng Jiang, Ju Wang, Peimin Deng, and Suqin Tang. Reasoning within expressive fuzzy rough description logics. *Fuzzy Sets and Systems*, 160(23):3403–3424, 2009.

[148] Yuncheng Jiang, Ju Wang, Suqin Tang, and Bao Xiao. Reasoning with rough description logics: An approximate concepts approach. *Information Sciences*, 179(5):600–612, 2009.

[149] Dazhou Kang, Baowen Xu, Jianjiang Lu, and Yanhui Li. Reasoning for a fuzzy description logic with comparison expressions. In *Proceedings of the International Workshop on Description Logics (DL-06)*. CEUR Workshop Proceedings, 2006.

[150] M.A. Khamsi and D. Misane. Disjunctive signed logic programs. *Fundamenta Informaticae*, 32:349–357, 1996.

[151] M.A. Khamsi and D. Misane. Fixed point theorems in logic programming. *Annals of Mathematics and Artificial Intelligence*, 21:231–243, 1997.

[152] M. Kifer and Ai Li. On the semantics of rule-based expert systems with uncertainty. In *Proceedings of the International Conference on Database Theory (ICDT-88)*, volume 326 of *Lecture Notes in Computer Science*, pages 102–117. Springer-Verlag, 1988.

[153] Michael Kifer and V.S. Subrahmanian. Theory of generalized annotated logic programming and its applications. *Journal of Logic Programming*, 12:335–367, 1992.

[154] Frank Klawonn and Rudolf Kruse. A Łukasiewicz logic based Prolog. *Mathware & Soft Computing*, 1(1):5–29, 1994.

[155] Erich Peter Klement, Radko Mesiar, and Endre Pap. *Triangular Norms*. Trends in Logic - Studia Logica Library. Kluwer Academic Publishers, 2000.

[156] George J. Klir and Bo Yuan. *Fuzzy sets and fuzzy logic: theory and applications*. Prentice-Hall, Inc., Upper Saddle River, NJ, USA, 1995.

[157] Stasinos Konstantopoulos and Georgios Apostolikas. Fuzzy-dl reasoning over unknown fuzzy degrees. In *Proceedings of the 2007 OTM Confederated international conference on On the move to meaningful internet systems - Volume Part II*, OTM-07, pages 1312–1318, Berlin, Heidelberg, 2007. Springer-Verlag.

[158] Stasinos Konstantopoulos, Vangelis Karkaletsis, and Dimitris Bilidas. An intelligent authoring environment for abstract semantic representations of cultural object descriptions. In *Proceedings of the EACL 2009 Workshop on Language Technology and Resources for Cultural Heritage, Social Sciences, Humanities, and Education (LaTeCHSHELT&R 2009)*, pages 10–17, 2009.

[159] Peter Kulmann and Sandra Sandri. An annotaded logic theorem prover for an extended possibilistic logic. *Fuzzy Sets and Systems*, 144:67–91, 2004.

[160] Laks Lakshmanan. An epistemic foundation for logic programming with uncertainty. In *Foundations of Software Technology and Theoretical Computer Science*, volume 880 of *Lecture Notes in Computer Science*, pages 89–100. Springer-Verlag, 1994.

[161] Laks V.S. Lakshmanan and Fereidoon Sadri. Uncertain deductive databases: a hybrid approach. *Information Systems*, 22(8):483–508, 1997.

[162] Laks V.S. Lakshmanan and Nematollaah Shiri. A parametric approach to deductive databases with uncertainty. *IEEE Transactions on Knowledge and Data Engineering*, 13(4):554–570, 2001.

[163] Chang-Shing Lee, Zhi-Wei Jian, and Lin-Kai Huang. A fuzzy ontology and its application to news summarization. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, 35(5):859–880, 2005.

[164] Chang-Shing Lee, M. H. Wang, and H. Hagras. A type-2 fuzzy ontology and its application to personal diabetic-diet recommendation. *IEEE Transactions on Fuzzy Systems*, 18(2):374–395, 2010.

[165] Ioan A. Letia and Adrian Groza. Modelling imprecise arguments in description logic. *Advances in Electrical and Computer Engineering*, 9(3):94–99, 2009.

[166] Francesca A. Lisi and Umberto Straccia. A logic-based computational method for the automated induction of fuzzy ontology axioms. *Fundamenta Informaticae*, 124(4):503–519, 2013.

[167] Francesca A. Lisi and Umberto Straccia. A system for learning GCI axioms in fuzzy description logics. In *Proceedings of the 26th International Workshop on Description Logics (DL-13)*, volume 1014 of *CEUR Workshop Proceedings*, pages 760–778. CEUR-WS.org, 2013.

[168] Francesca A. Lisi and Umberto Straccia. Can ilp deal with incomplete and vague structured knowledge? In Stephen H. Muggleton and Hiroaki Watanabe, editors, *Latest Advances in Inductive Logic Programming*, chapter 21, pages 199–206. World Scientific, 2014.

[169] Francesca A. Lisi and Umberto Straccia. Learning in description logics with fuzzy concrete domains. *Fundamenta Informaticae*, 140(3-4):373–391, 2015.

[170] Francesca Alessandra Lisi and Umberto Straccia. An inductive logic programming approach to learning inclusion axioms in fuzzy description logics. In *26th Italian Conference on Computational Logic (CILC-11)*, volume 810, pages 57–71. CEUR Electronic Workshop Proceedings, 2011.

[171] Francesca Alessandra Lisi and Umberto Straccia. Towards learning fuzzy dl inclusion axioms. In *9th International Workshop on Fuzzy Logic and Applications (WILF-11)*, volume 6857 of *Lecture Notes in Computer Science*, pages 58–66, Berlin, 2011. Springer Verlag.

[172] Francesca Alessandra Lisi and Umberto Straccia. Dealing with incompleteness and vagueness in inductive logic programming. In *28th Italian Conference on Computational Logic (CILC-13)*, volume 1068, pages 179–193. CEUR Electronic Workshop Proceedings, 2013.

[173] Francesca Alessandra Lisi and Umberto Straccia. A foil-like method for learning under incompleteness and vagueness. In *23rd International Conference on Inductive Logic Programming*, volume 8812 of *Lecture Notes in Artificial Intelligence*, pages 123–139, Berlin, 2014. Springer Verlag. Revised Selected Papers.

[174] Chunchen Liu, Dayou Liu, and Shengsheng Wang. Situation modeling and identifying under uncertainty. In *Proceedings of the 2nd Pacific-Asia Conference on Circuits, Communications and System (PACCS 2010)*, pages 296–299, 2010.

[175] Chunchen Liu, Dayou Liu, and Shengsheng Wang. Fuzzy geospatial information modeling in geospatial semantic retrieval. *Advances in Mathematical and Computational Methods*, 2(4):47–53, 2012.

[176] Ou Liu, Qijian Tian, and Jian Ma. A fuzzy description logic approach to model management in R&D project selection. In *Proceedings of the 8th Pacific Asia Conference on Information Systems (PACIS-04)*, 2004.

[177] John W. Lloyd. *Foundations of Logic Programming.* Springer, Heidelberg, RG, 1987.

[178] Nuno Lopes, Axel Polleres, Umberto Straccia, and Antoine Zimmermann. AnQL: SPARQLing up annotated RDF. In *Proceedings of the International Semantic Web Conference (ISWC-10)*, volume 6496 of *Lecture Notes in Computer Science*, pages 518–533. Springer-Verlag, 2010.

[179] Nuno Lopes, Antoine Zimmermann, Aidan Hogan, Gergely Lukacsy, Axel Polleres, Umberto Straccia, and Stefan Decker. Rdf needs annotations. In *Proceedings of W3C Workshop — RDF Next Steps*, `http://www.w3.org/2009/12/rdf-ws/`, 2010.

[180] Yann Loyer and Umberto Straccia. Uncertainty and partial non-uniform assumptions in parametric deductive databases. In *Proceedings of the 8th European Conference on Logics in Artificial Intelligence (JELIA-02)*, volume 2424 of *Lecture Notes in Computer Science*, pages 271–282, Cosenza, Italy, 2002. Springer-Verlag.

[181] Yann Loyer and Umberto Straccia. The well-founded semantics in normal logic programs with uncertainty. In *Proceedings of the 6th International Symposium on Functional and Logic Programming (FLOPS-2002)*, volume 2441 of *Lecture Notes in Computer Science*, pages 152–166, Aizu, Japan, 2002. Springer-Verlag.

[182] Yann Loyer and Umberto Straccia. The approximate well-founded semantics for logic programs with uncertainty. In *28th International Symposium on Mathematical Foundations of Computer Science (MFCS-2003)*, volume 2747 of *Lecture Notes in Computer Science*, pages 541–550, Bratislava, Slovak Republic, 2003. Springer-Verlag.

[183] Yann Loyer and Umberto Straccia. Default knowledge in logic programs with uncertainty. In *Proceedings of the 19th International Conference on Logic Programming (ICLP-03)*, volume 2916 of *Lecture Notes in Computer Science*, pages 466–480, Mumbai, India, 2003. Springer Verlag.

[184] Yann Loyer and Umberto Straccia. Epistemic foundation of the well-founded semantics over bilattices. In *29th International Symposium on Mathematical Foundations of Computer Science (MFCS-2004)*, volume 3153 of *Lecture Notes in Computer Science*, pages 513–524, Bratislava, Slovak Republic, 2004. Springer Verlag.

[185] Yann Loyer and Umberto Straccia. Any-world assumptions in logic programming. *Theoretical Computer Science*, 342(2-3):351–381, 2005.

[186] Yann Loyer and Umberto Straccia. Epistemic foundation of stable model semantics. *Journal of Theory and Practice of Logic Programming*, 6:355–393, 2006.

[187] James J. Lu. Logic programming with signs and annotations. *Journal of Logic and Computation*, 6(6):755–778, 1996.

[188] James J. Lu, Jacques Calmet, and Joachim Schü. Computing multiple-valued logic programs. *Mathware & Soft Computing*, 2(4):129–153, 1997.

[189] Thomas Lukasiewicz. Fuzzy description logic programs under the answer set semantics for the semantic web. In *Second International Conference on Rules and Rule Markup Languages for the Semantic Web (RuleML-06)*, pages 89–96. IEEE Computer Society, 2006.

[190] Thomas Lukasiewicz. Fuzzy description logic programs under the answer set semantics for the semantic web. *Fundamenta Informaticae*, 82(3):289–310, 2008.

[191] Thomas Lukasiewicz and Umberto Straccia. Description logic programs under probabilistic uncertainty and fuzzy vagueness. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU-07)*, volume 4724 of *Lecture Notes in Computer Science*, pages 187–198. Springer Verlag, 2007.

[192] Thomas Lukasiewicz and Umberto Straccia. Tightly integrated fuzzy description logic programs under the answer semantics for the semantic web. In *Proceedings of the First International Conference on Web Reasoning and Rule Systems (RR-07)*, volume 4524 of *Lecture Notes in Computer Science*, pages 289—298. Springer Verlag, 2007.

[193] Thomas Lukasiewicz and Umberto Straccia. Tightly integrated fuzzy description logic programs under the answer semantics for the semantic web. INFSYS RESEARCH REPORT 1843-07-03, INSTITUT FüR INFORMATIONSSYSTEME ARBEITSBEREICH WISSENSBASIERTE SYSTEME, Technische Universität Wien, 2007.

[194] Thomas Lukasiewicz and Umberto Straccia. Top-k retrieval in description logic programs under vagueness for the semantic web. In *Proceedings of the 1st International Conference on Scalable Uncertainty Management (SUM-07)*, volume 4772 of *Lecture Notes in Computer Science*, pages 16–30. Springer Verlag, 2007.

[195] Thomas Lukasiewicz and Umberto Straccia. Managing uncertainty and vagueness in description logics for the semantic web. *Journal of Web Semantics*, 6:291–308, 2008.

[196] Thomas Lukasiewicz and Umberto Straccia. Tightly coupled fuzzy description logic programs under the answer set semantics for the semantic web. *International Journal on Semantic Web and Information Systems*, 4(3):68–89, 2008.

[197] Thomas Lukasiewicz and Umberto Straccia. Description logic programs under probabilistic uncertainty and fuzzy vagueness. *International Journal of Approximate Reasoning*, 50(6):837–853, 2009.

[198] Thomas Lukasiewicz and Umberto Straccia. Tightly integrated fuzzy description logic programs under the answer semantics for the semantic web. In Miltiadis Lytras & Amith Sheth, editor, *Progressive Concepts for Semantic Web Evolution: Applications and Developments*, chapter 11, pages 237–256. IGI Global, 2010.

[199] Nicolas Madrid and Umberto Straccia. Towards a top-k retrieval for non-monotonic ranking functions. In *In Proceedings of the 10th International Conference on Flexible Query Answering Systems (FQAS-13)*, volume 8132 of *Lecture Notes in Artificial Intelligence*, pages 507–518. Springer Verlag, 2013.

[200] P. Magrez and P. Smets. Fuzzy modus ponens: a new model suitable for applications in knowledge-based systems. *International Journal of Intelligent Systems*, 4:181–200, 1989.

[201] Theofilos P. Mailis, Giorgos Stoilos, and Giorgos B. Stamou. Expressive reasoning with horn rules and fuzzy description logics. In *Proceedings of the First International Conference on Web Reasoning and Rule Systems (RR-07)*, volume 4524 of *Lecture Notes in Computer Science*, pages 43–57, 2007.

[202] Zoran Majkic. Coalgebraic semantics for logic programs. In *18th Workshop on (Constraint) Logic Programming (WCLP-05)*, Ulm, Germany, 2004.

[203] Zoran Majkic. Many-valued intuitionistic implication and inference closure in a bilattice-based logic. In *35th International Symposium on Multiple-Valued Logic (ISMVL-05)*, pages 214–220, 2005.

[204] Zoran Majkic. Truth and knowledge fixpoint semantics for many-valued logic programming. In *19th Workshop on (Constraint) Logic Programming (WCLP-05)*, pages 76–87, Ulm, Germany, 2005.

[205] V. W. Marek and M. Truszczyński. Logic programming with costs. Technical report, University of Kentucky, 2000. Available at ftp://al.cs.engr.uky.edu/cs/manuscripts/lp-costs.ps.

[206] Draltan Marin. A formalization of rdf. Technical Report TR/DCC-2006-8, Deptartment of Computer Science, Universidad de Chile, http://www.dcc.uchile.cl/cgutierr/ftp/draltan.pdf, 2004.

[207] T. P. Martin, J. F. Baldwin, and B. W. Pilsworth. The implementation of FProlog –a fuzzy prolog interpreter. *Fuzzy Sets and Systems*, 23(1):119–129, 1987.

[208] Carmen Martínez-Cruz, Albert van der Heide, Daniel Sánchez, and Gracian Triviño. An approximation to the computational theory of perceptions using ontologies. *Expert Systems with Applications*, 39(10):9494–9503, 2012.

[209] Cristinel Mateis. Extending disjunctive logic programming by t-norms. In *Proceedings of the 5th International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR-99)*, volume 1730 of *Lecture Notes in Computer Science*, pages 290–304. Springer-Verlag, 1999.

[210] Cristinel Mateis. Quantitative disjunctive logic programming: Semantics and computation. *AI Communications*, 13:225–248, 2000.

[211] Jesús Medina and Manuel Ojeda-Aciego. Multi-adjoint logic programming. In *Proceedings of the 10th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-04)*, pages 823–830, 2004.

[212] Jesús Medina, Manuel Ojeda-Aciego, and Peter Vojtás. Multi-adjoint logic programming with continuous semantics. In *Proceedings of the 6th International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR-01)*, volume 2173 of *Lecture Notes in Artificial Intelligence*, pages 351–364. Springer Verlag, 2001.

[213] Jesús Medina, Manuel Ojeda-Aciego, and Peter Vojtás. A procedural semantics for multi-adjoint logic programming. In *Proceedings of the 10th Portuguese Conference on Artificial Intelligence on Progress in Artificial Intelligence, Knowledge Extraction, Multi-Agent Systems, Logic Programming and Constraint Solving*, pages 290–297. Springer-Verlag, 2001.

[214] Jesús Medina, Manuel Ojeda-Aciego, and Peter Vojtás. Similarity-based unification: a multi-adjoint approach. *Fuzzy sets and systems*, 1(146):43–62, 2004.

[215] Carlo Meghini, Fabrizio Sebastiani, and Umberto Straccia. A model of multimedia information retrieval. *Journal of the ACM*, 48(5):909–970, 2001.

[216] P. S. Mostert and A. L. Shields. On the structure of semigroups on a compact manifold with boundary. *Annals of Mathematics*, 65:117–143, 1957.

[217] M. Mukaidono. Foundations of fuzzy logic programming. In *Advances in Fuzzy Systems – Application and Theory*, volume 1. World Scientific, Singapore, 1996.

[218] M. Mukaidono, Z. Shen, and L. Ding. Fundamentals of fuzzy prolog. *International Journal of Approximate Reasoning*, 3(2):179–193, 1989.

[219] Sergio Muñoz, Jorge Pérez, and Claudio Gutiérrez. Minimal deductive systems for RDF. In *4th European Semantic Web Conference (ESWC-07)*, volume 4519 of *Lecture Notes in Computer Science*, pages 53–67. Springer Verlag, 2007.

[220] OWL Web Ontology Language overview. *http://www.w3.org/TR/owl-features/*. W3C, 2004.

[221] OWL 2 Web Ontology Language Document Overview. *https://www.w3.org/TR/owl2-overview/*. W3C, 2009.

[222] Jeff Z. Pan, Giorgos Stamou, Giorgos Stoilos, and Edward Thomas. Expressive querying over fuzzy DL-Lite ontologies. In *Twentieth International Workshop on Description Logics*, 2007.

[223] Leonard Paulik. Best possible answer is computable for fuzzy SLD-resolution. In Petr Hajék, editor, *Gödel 96: Logical Foundations of Mathematics, Computer Science, and Physics*, volume 6 of *Lecture Notes in Logic*, pages 257–266. Springer Verlag, 1996.

[224] Ignacio J. Pérez, Robin Wikström, József Mezei, Christer Carlsson, and Enrique Herrera-Viedma. A new consensus model for group decision making using fuzzy ontology. *Soft Computing*, 17(9):1617–1627, 2013.

[225] Thanh Tho Quan, Siu Cheung Hui, and Alvis Cheuk M. Fong. Automatic fuzzy ontology generation for semantic help-desk support. *IEEE Transactions on Industrial Informatics*, 2(3):155–164, 2006.

[226] Thanh Tho Quan, Siu Cheung Hui, Alvis Cheuk M. Fong, and Tru Hoang Cao. Automatic fuzzy ontology generation for semantic web. *IEEE Transactions on Knowledge and Data Engineering*, 18(6):842–856, 2006.

[227] Azzurra Ragone, Umberto Straccia, Fernando Bobillo, Tommaso Di Noia, and Eugenio Di Sciascio. Fuzzy bilateral matchmaking in e-marketplaces. In *12th International Conference on Knowledge-Based & Intelligent Information & Engineering Systems - KES2008*, volume 5179 of *Lecture Notes in Artificial Intelligence*, pages 293–301. Springer, 2008.

[228] Azzurra Ragone, Umberto Straccia, Tommaso Di Noia, Eugenio Di Sciascio, and Francesco M. Donini. Extending datalog for matchmaking in p2p e-marketplaces. In Michelangelo Ceci, Donato Malerba, and Letizia Tanca, editors, *15th Italian Symposium on Advanced Database Systems (SEBD-07)*, pages 463–470, 2007.

[229] Azzurra Ragone, Umberto Straccia, Tommaso Di Noia, Eugenio Di Sciascio, and Francesco M. Donini. Vague knowledge bases for matchmaking in p2p e-marketplaces. In *4th European Semantic Web Conference (ESWC-07)*, volume 4519 of *Lecture Notes in Computer Science*, pages 414–428. Springer Verlag, 2007.

[230] Azzurra Ragone, Umberto Straccia, Tommaso Di Noia, Eugenio Di Sciascio, and Francesco M. Donini. Towards a fuzzy logic for automated multi-issue negotiation. In *Proceedings of the 5th International Symposium on Foundations of Information and Knowledge Systems (FoIKS-08)*, volume 4932 of *Lecture Notes in Computer Science*, pages 381–396. Springer Verlag, 2008.

[231] Azzurra Ragone, Umberto Straccia, Tommaso Di Noia, Eugenio Di Sciascio, and Francesco M. Donini. Fuzzy matchmaking in e-marketplaces of peer entities using Datalog. *Fuzzy Sets and Systems*, 160(2):251–268, 2009.

[232] RDF Semantics. *http://www.w3.org/TR/rdf-mt/*. W3C, 2004.

[233] Paul C. Rhodes and Sabah Merad Menani. Towards a fuzzy logic programming system: a clausal form fuzzy logic. *Knowledge-Based Systems*, 8(4):174–182, 1995.

[234] James A. Rodger. A fuzzy linguistic ontology payoff method for aerospace real options valuation. *Expert Systems with Applications*, 40(8), 2013.

[235] Natalia Díaz Rodríguez, M. P. Cuéllar, Johan Lilius, and Miguel Delgado Calvo-Flores. A survey on ontologies for human behavior recognition. *ACM Computing Surveys*, 46(4):43:1–43:33, March 2014.

[236] William C. Rounds and Guo-Qiang Zhang. Clausal logic and logic programming in algebraic domains. *Information and Computation*, 171:183–200, 2001.

[237] Rule Interchange Format (RIF). *http://www.w3.org/2001/sw/wiki/RIF*. W3C, 2011.

[238] Daniel Sanchez and Andrea G.B. Tettamanzi. Generalizing quantification in fuzzy description logics. In *Proceedings 8th Fuzzy Days in Dortmund*, 2004.

[239] Daniel Sanchez and Andrea G.B. Tettamanzi. Reasoning and quantification in fuzzy description logics. In *International Workshop on Fuzzy Logic and Applications (WILF-05)*, volume 3849 of *Lecture Notes in Artificial Intelligence*, pages 81–88, 2005.

[240] Daniel Sanchez and Andrea G.B. Tettamanzi. Fuzzy quantification in fuzzy description logics. In Elie Sanchez, editor, *Capturing Intelligence: Fuzzy Logic and the Semantic Web*. Elsevier, 2006.

[241] Elie Sanchez, editor. *Fuzzy Logic and the Semantic Web*, volume 1 of *Capturing Intelligence*. Elsevier Science, 2006.

[242] Michael Schroeder and Ralf Schweimeier. Fuzzy argumentation and extended logic programming. In *Proceedings of ECSQARU Workshop Adventures in Argumentation*, 2001.

[243] Michael Schroeder and Ralf Schweimeier. Arguments and misunderstandings: Fuzzy unification for negotiating agents. In *Proceedings of the ICLP Workshop CLIMA02*. Elsevier, 2002.

[244] Michael Schroeder and Ralf Schweimeier. Fuzzy unification and argumentation for well-founded semantics. In *Proceedings of the Conference on Current Trends in Theory and Practice of Informatics (SOFSEM-04)*, volume 2932 of *Lecture Notes in Computer Science*, pages 102–121. Springer Verlag, 2004.

[245] Maria I. Sessa. Approximate reasoning by similarity-based sld resolution. *Theoretical Computer Science*, 275:389–426, 2002.

[246] Ehud Y. Shapiro. Logic programs with uncertainties: A tool for implementing rule-based systems. In *Proceedings of the 8th International Joint Conference on Artificial Intelligence (IJCAI-83)*, pages 529–532, 1983.

[247] Zuliang Shen, Liya Ding, and Masao Mukaidono. *Fuzzy Computing*, chapter A Theoretical Framework of Fuzzy Prolog Machine, pages 89–100. Elsevier Science Publishers B.V., 1988.

[248] Václav Slavícek. An ontology-driven fuzzy workflow system. In *Proceedings of the 39th International Conference on Current Trends in Theory and Practice of Computer Science (SOFSEM 2013)*, volume 7741 of *Lecture Notes in Computer Science*, pages 515–527. Springer, 2013.

[249] Rafael Peñaloza Stefan Borgwardt. Consistency reasoning in lattice-based fuzzy description logics. *International Journal of Approximate Reasoning*, 2014. To appear.

[250] G. Stoilos, G. Stamou, and J. Z. Pan. Fuzzy extensions of OWL: Logical properties and reduction to fuzzy description logics. *International Journal of Approximate Reasoning*, 51(6):656–679, July 2010.

[251] George Stoilos and Giorgos Stamou. Extending fuzzy description logics for the semantic web. In *3rd International Workshop of OWL: Experiences and Directions*, 2007.

[252] George Stoilos, Giorgos Stamou, Jeff Pan, Vassilis Tzouvaras, and Ian Horrocks. The fuzzy description logic f-SHIN. In *International Workshop on Uncertainty Reasoning For the Semantic Web*, 2005.

[253] Giorgos Stoilos, Nikolaos Simou, Giorgos Stamou, and Stefanos Kollias. Uncertainty and the semantic web. *IEEE Intelligent Systems*, 21(5):84–87, 2006.

[254] Giorgos Stoilos, Giorgos Stamou, Vassilis Tzouvaras, Jeff Z. Pan, and Ian Horrock. A Fuzzy Description Logic for Multimedia Knowledge Representation. In *Proceedings of the International Workshop on Multimedia and the Semantic Web*, 2005.

[255] Giorgos Stoilos, Giorgos B. Stamou, Jeff Z. Pan, Vassilis Tzouvaras, and Ian Horrocks. Reasoning with very expressive fuzzy description logics. *Journal of Artificial Intelligence Research*, 30:273–320, 2007.

[256] Giorgos Stoilos, Umberto Straccia, Giorgos Stamou, and Jeff Z. Pan. General concept inclusions in fuzzy description logics. In *Proceedings of the 17th Eureopean Conference on Artificial Intelligence (ECAI-06)*, pages 457–461. IOS Press, 2006.

[257] Umberto Straccia. A fuzzy description logic. In *Proceedings of the 15th National Conference on Artificial Intelligence (AAAI-98)*, pages 594–599, Madison, USA, 1998.

[258] Umberto Straccia. *Foundations of a Logic Based Approach to Multimedia Document Retrieval.* PhD thesis, Department of Computer Science, University of Dortmund, Dortmund, Germany, June 1999.

[259] Umberto Straccia. A framework for the retrieval of multimedia objects based on four-valued fuzzy description logics. In F. Crestani and Gabriella Pasi, editors, *Soft Computing in Information Retrieval: Techniques and Applications*, pages 332–357. Physica Verlag (Springer Verlag), Heidelberg, Germany, 2000.

[260] Umberto Straccia. Reasoning within fuzzy description logics. *Journal of Artificial Intelligence Research*, 14:137–166, 2001.

[261] Umberto Straccia. Transforming fuzzy description logics into classical description logics. In *Proceedings of the 9th European Conference on Logics in Artificial Intelligence (JELIA-04)*, volume 3229 of *Lecture Notes in Computer Science*, pages 385–399, Lisbon, Portugal, 2004. Springer Verlag.

[262] Umberto Straccia. Uncertainty in description logics: a lattice-based approach. In *Proceedings of the 10th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-04)*, pages 251–258, 2004.

[263] Umberto Straccia. Description logics with fuzzy concrete domains. In Fahiem Bachus and Tommi Jaakkola, editors, *21st Conference on Uncertainty in Artificial Intelligence (UAI-05)*, pages 559–567, Edinburgh, Scotland, 2005. AUAI Press.

[264] Umberto Straccia. Fuzzy alc with fuzzy concrete domains. In *Proceeedings of the International Workshop on Description Logics (DL-05)*, volume 147, pages 96–103, Edinburgh, Scotland, 2005. CEUR-WS.org.

[265] Umberto Straccia. Query answering in normal logic programs under uncertainty. In *8th European Conferences on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU-05)*, volume 3571 of *Lecture Notes in Computer Science*, pages 687–700, Barcelona, Spain, 2005. Springer Verlag.

[266] Umberto Straccia. Uncertainty management in logic programming: Simple and effective top-down query answering. In Rajiv Khosla, Robert J. Howlett, and Lakhmi C. Jain, editors, *9th International Conference on Knowledge-Based & Intelligent Information & Engineering Systems (KES-05), Part II*, volume 3682 of *Lecture Notes in Computer Science*, pages 753–760, Melbourne, Australia, 2005. Springer Verlag.

[267] Umberto Straccia. Annotated answer set programming. In *Proceedings of the 11th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-06)*, pages 1212–1219. E.D.K., Paris, 2006.

[268] Umberto Straccia. Answering vague queries in fuzzy DL-Lite. In *Proceedings of the 11th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-06)*, pages 2238–2245. E.D.K., Paris, 2006.

[269] Umberto Straccia. Description logics over lattices. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 14(1):1–16, 2006.

[270] Umberto Straccia. Fuzzy description logic programs. In *Proceedings of the 11th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-06)*, pages 1818–1825. E.D.K., Paris, 2006.

[271] Umberto Straccia. Query answering under the any-world assumption for normal logic programs. In *Proceedings of the 10th International Conference on Principles of Knowledge Representation (KR-06)*, pages 329–339. AAAI Press, 2006.

[272] Umberto Straccia. Towards top-k query answering in deductive databases. In *Proceedings of the 2006 IEEE International Conference on Systems, Man and Cybernetics (SMC-06)*, pages 4873–4879. IEEE, 2006.

[273] Umberto Straccia. Towards top-k query answering in description logics: the case of DL-Lite. In *Proceedings of the 10th European Conference on Logics in Artificial Intelligence (JELIA-06)*, volume 4160 of *Lecture Notes in Computer Science*, pages 439–451, Liverpool, UK, 2006. Springer Verlag.

[274] Umberto Straccia. Uncertainty and description logic programs over lattices. In Elie Sanchez, editor, *Fuzzy Logic and the Semantic Web*, Capturing Intelligence, chapter 7, pages 115–133. Elsevier, 2006.

[275] Umberto Straccia. Reasoning in Ł-$\mathcal{SHIF}$: an expressive fuzzy description logic under Łukasiewicz semantics. Technical Report TR-2007-10-18, Istituto di Scienza e Tecnologie dell'Informazione, Consiglio Nazionale delle Ricerche, Pisa, Italy, 2007.

[276] Umberto Straccia. A top-down query answering procedure for normal logic programs under the any-world assumption. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU-07)*, volume 4724 of *Lecture Notes in Computer Science*, pages 115–127. Springer Verlag, 2007.

[277] Umberto Straccia. Towards vague query answering in logic programming for logic-based information retrieval. In *World Congress of the International Fuzzy Systems Association (IFSA-07)*, volume 4529 of *Lecture Notes in Computer Science*, pages 125–134, Cancun, Mexico, 2007. Springer Verlag.

[278] Umberto Straccia. Fuzzy description logic programs. In C. Marsala B. Bouchon-Meunier, R.R. Yager and M. Rifqi, editors, *Uncertainty and Intelligent Information Systems*, chapter 29, pages 405–418. World Scientific, 2008.

[279] Umberto Straccia. Managing uncertainty and vagueness in description logics, logic programs and description logic programs. In *Reasoning Web, 4th International Summer School, Tutorial Lectures*, volume 5224 of *Lecture Notes in Computer Science*, pages 54–103. Springer Verlag, 2008.

[280] Umberto Straccia. A minimal deductive system for general fuzzy RDF. In *Proceedings of the 3rd International Conference on Web Reasoning and Rule Systems (RR-09)*, volume 5837 of *Lecture Notes in Computer Science*, pages 166–181. Springer-Verlag, 2009.

[281] Umberto Straccia. Multi-criteria decision making in fuzzy description logics: A first step. In *13th International Conference on Knowledge-Based & Intelligent Information & Engineering Systems - KES-09*, volume 5711 of *Lecture Notes in Artificial Intelligence*, pages 79–87. Springer, 2009.

[282] Umberto Straccia. Towards spatial reasoning in fuzzy description logics. In *2009 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE-09)*, pages 512–517. IEEE Computer Society, 2009.

[283] Umberto Straccia. An ontology mediated multimedia information retrieval system. In *Proceedings of the the 40th International Symposium on Multiple-Valued Logic (ISMVL-10)*, pages 319–324. IEEE Computer Society, 2010.

[284] Umberto Straccia. Softfacts: A top-k retrieval engine for ontology mediated access to relational databases. In *Proceedings of the 2010 IEEE International Conference on Systems, Man and Cybernetics (SMC-10)*, pages 4115–4122. IEEE Press, 2010.

[285] Umberto Straccia. *Foundations of Fuzzy Logic and Semantic Web Languages.* CRC Studies in Informatics Series. Chapman & Hall, 2013.

[286] Umberto Straccia. On the top-k retrieval problem for ontology-based access to databases. In Sławomir Pivert, Olivier; Zadrożny, editor, *Flexible Approaches in Data, Information and Knowledge Management*, volume 497 of *Studies in Computational Intelligence*, chapter 5, pages 95–114. Springer Verlag, 2014.

[287] Umberto Straccia. All about fuzzy description logics and applications. In *Reasoning Web, 11th International Summer School, Tutorial Lectures*, volume 9203 of *Lecture Notes in Computer Science*, pages 1–31. Springer Verlag, 2015.

[288] Umberto Straccia. Fuzzy semantic web languages and beyond. In *Invited Talk at the 30th International Conference on Industrial, Engineering, Other Applications of Applied Intelligent Systems (IEA/AIE 2017)*, volume 10350 of *Lecture Notes in Artificial Intelligence*, pages 3–8. Springer Verlag, 2017.

[289] Umberto Straccia and Fernando Bobillo. Mixed integer programming, general concept inclusions and fuzzy description logics. In *Proceedings of the 5th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT-07)*, volume 2, pages 213–220, Ostrava, Czech Republic, 2007. University of Ostrava.

[290] Umberto Straccia and Fernando Bobillo. Mixed integer programming, general concept inclusions and fuzzy description logics. *Mathware & Soft Computing*, 14(3):247–259, 2007.

[291] Umberto Straccia and Fernando Bobillo. From fuzzy to annotated semantic web languages. In *Reasoning Web: Logical Foundation of Knowledge Graph Construction and Query Answering, 12th International Summer School 2016, Tutorial Lectures.*, volume 9885 of *Lecture Notes in Computer Science*, pages 203—240. Springer Verlag, 2017.

[292] Umberto Straccia, Nuno Lopes, Gergely Lukacsy, and Axel Polleres. A general framework for representing and reasoning with annotated semantic web data. In *Proceedings of the 24th AAAI Conference on Artificial Intelligence (AAAI-10)*, pages 1437–1442. AAAI Press, 2010.

[293] Umberto Straccia and Nicolas Madrid. A top-k query answering procedure for fuzzy logic programming. *Fuzzy Sets and Systems*, 205:1–29, 2012.

[294] Umberto Straccia and Matteo Mucci. pFOIL-DL: Learning (fuzzy) $\mathcal{EL}$ concept descriptions from crisp OWL data using a probabilistic ensemble estimation. In *Proceedings of the 30th Annual ACM Symposium on Applied Computing (SAC-15)*, pages 345–352, Salamanca, Spain, 2015. ACM.

[295] Umberto Straccia, Manuel Ojeda-Aciego, and Carlos V. Damásio. On fixed-points of multi-valued functions on complete lattices and their application to generalized logic programs. *SIAM Journal on Computing*, 8(5):1881–1911, 2009.

[296] Umberto Straccia, Eufemia Tinelli, Tommaso Di Noia, Eugenio Di Sciascio, and Simona Colucci. Semantic-based top-k retrieval for competence management. In J. Rauch et al., editor, *Proceedings of the 18th International Symposium on Methodologies for Intelligent Systems (ISMIS-09)*, volume 5722 of *Lecture Notes in Artificial Intelligence*, pages 473–482. Springer Verlag, 2009.

[297] Umberto Straccia, Eufemia Tinelli, Tommaso Di Noia, Eugenio Di Sciascio, and Simona Colucci. Top-k retrieval for automated human resource management. In *Proceedings of the 17th Italian Symposium on Advanced Database Systems (SEBD-09)*, pages 161–168, 2009.

[298] Umberto Straccia and Giulio Visco. DL-Media: an ontology mediated multimedia information retrieval system. In *Proceeedings of the International Workshop on Description Logics (DL-07)*, volume 250, Insbruck, Austria, 2007. CEUR.

[299] Umberto Straccia and Giulio Visco. DLMedia: an ontology mediated multimedia information retrieval system. In *Proceedings of the Fourth International Workshop on Uncertainty Reasoning for the Semantic Web, Karlsruhe, Germany, October 26, (URSW-08)*, volume 423 of *CEUR Workshop Proceedings*. CEUR-WS.org, 2008.

[300] V.S. Subramanian. On the semantics of quantitative logic programs. In *Proc. 4th IEEE Symp. on Logic Programming*, pages 173–182. Computer Society Press, 1987.

[301] Konstantin Todorov, Céline Hudelot, Adrian Popescu, and Peter Geibel. Fuzzy ontology alignment using background knowledge. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 22(1):75–112, 2014.

[302] C. Tresp and R. Molitor. A description logic for vague knowledge. In *Proceedings of the 13th European Conference on Artificial Intelligence (ECAI-98)*, Brighton (England), August 1998.

[303] Hudson Turner. Signed logic programs. In Maurice Bruynooghe, editor, *Logic Programming: Proc. of the 1994 International Symposium*, pages 61–75. The MIT Press, 1994.

[304] J. D. Ullman. *Principles of Database and Knowledge Base Systems*, volume 1,2. Computer Science Press, Potomac, Maryland, 1989.

[305] M.H. van Emden. Quantitative deduction and its fixpoint theory. *Journal of Logic Programming*, 4(1):37–53, 1986.

[306] T. Venetis, G. Stoilos, G. Stamou, and S. Kollias. f-DLPs: Extending description logic programs with fuzzy sets and fuzzy logic. In *IEEE International Conference on Fuzzy Systems (Fuzz-IEEE 2007)*, 2007.

[307] Peter Vojtás. Fuzzy logic programming. *Fuzzy Sets and Systems*, 124:361–370, 2001.

[308] Peter Vojtás and Leonard Paulík. Soundness and completeness of non-classical extended SLD-resolution. In *5th International Workshop on Extensions of Logic Programming (ELP'96)*, volume 1050 of *Lecture Notes in Artificial Intelligence*, pages 289–301, Leipzig, Germany, 1996.

[309] Peter Vojtás and Marta Vomlelová. Transformation of deductive and inductive tasks between models of logic programming with imperfect information. In *Proceedings of the 10th International Conference on Information Processing and Managment of Uncertainty in Knowledge-Based Systems, (IPMU-04)*, pages 839–846, 2004.

[310] Gerd Wagner. Negation in fuzzy and possibilistic logic programs. In T. Martin and F. Arcelli, editors, *Logic programming and Soft Computing*. Research Studies Press, 1998.

[311] Manolis Wallace. Ontologies and soft computing in flexible querying. *Control and Cybernetics*, 38(2):481–507, 2009.

[312] Hailong Wang, Z. M. Ma, and Junfu Yin. Fresg: A kind of fuzzy description logic reasoner. In *Proceedings of the 20th International Conference on Database and Expert Systems Applications*, DEXA '09, pages 443–450, Berlin, Heidelberg, 2009. Springer-Verlag.

[313] XML. `http://www.w3.org/XML/`. W3C.

[314] Cristiane A. Yaguinuma, Marilde T. P. Santos, Heloisa A. Camargo, and Marek Reformat. A fml-based hybrid reasoner combining fuzzy ontology and mamdani inference. In *Proceedings of the 22nd IEEE International Conference on Fuzzy Systems (FUZZ-IEEE 2013)*, 2013.

[315] Jianjiang Lu Yanhui Li, Baowen Xu and Dazhou Kang. Discrete tableau algorithms for $\mathcal{FSHI}$. In *Proceeedings of the International Workshop on Description Logics (DL-06)*. CEUR, 2006.

[316] H. Yasui, Y. Hamada, and M. Mukaidono. Fuzzy prolog based on lukasiewicz implication and bounded product. *IEEE Transactions on Fuzzy Systems*, 2:949–954, 1995.

[317] John Yen. Generalizing term subsumption languages to fuzzy logic. In *Proceedings of the 12th International Joint Conference on Artificial Intelligence (IJCAI-91)*, pages 472–477, Sydney, Australia, 1991.

[318] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965.

[319] Lei Zhang, Yong Yu, Jian Zhou, ChenXi Lin, and Yin Yang. An enhanced model for searching in semantic portals. In *WWW '05: Proceedings of the 14th international conference on World Wide Web*, pages 453–462, New York, NY, USA, 2005. ACM Press.

[320] Zhangquan Zhou, Guilin Qi, Chang Liu, Pascal Hitzler, and Raghava Mutharaju. Reasoning with fuzzy-$\mathcal{EL}^+$ ontologies using mapreduce. In *20th European Conference on Artificial Intelligence (ECAI-12)*, pages 933–934. IOS Press, 2012.

[321] Antoine Zimmermann, Nuno Lopes, Axel Polleres, and Umberto Straccia. A general framework for representing, reasoning and querying with annotated semantic web data. *Journal of Web Semantics*, 11:72–95, March 2012.